Strategic Experimentation with Asymmetric Information*

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Abstract

This paper studies strategic experimentation between two players, with one player initially better informed about the state of nature. They are otherwise symmetric, and observe past experimentation decisions and outcomes. I construct an equilibrium in which a mutual encouragement effect arises: as the public information becomes discouraging, the informed player's high effort continuously brings in good news, encouraging the uninformed player to experiment; in return, the uninformed player's experimentation pattern yields an increasing reward, encouraging the informed player to experiment. Due to this effect, players' total effort can increase over time, and the uninformed player may grow increasingly optimistic, despite the discouraging public information. Moreover, creating information asymmetry improves ex ante total welfare when the informed player's initial signal is sufficiently precise.

1 Introduction

Experimentation is an important mechanism through which agents discover new ideas and learn their value, thereby promoting technological change, and driving economic growth.[1] In

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[1] Endogenous technological change is a key driver of economic growth, as argued by endogenous growth theory (Romer, 1990; Aghion and Howitt, 1992). On the role of experimentation in the discovery and selection of new ideas, see Romer (1994, page 12), Nelson and Winter (1994, Chapter 11).

many environments, agents learn both from their own and from others' experimentations. For example, farmers learn from their own and others' experiences whether a new fertilizer improves yield (Foster and Rosenzweig, 1995; Conley and Udry, 2010); physicians learn through their own and others' prescriptions the efficacy of a new drug after it is approved by the FDA (Coleman, Katz and Menzel, 1957; Iyengar, Van den Bulte and Valente, 2011); firms in a strategic alliance learn from one another whether their newly developed product has a high demand.

In such environments, the information generated from experimentation is a public good. A free-riding problem naturally arises: agents experiment less than they would do if they acted cooperatively. The free-riding problem is well studied in symmetric information frameworks, for instance, by Bolton and Harris (1999), and Keller, Rady and Cripps (2005).[2] However, in many environments, initial information asymmetry is empirically relevant: some agent (a well educated farmer, a specialist physician, the designer of a new product) initially has better information about the value of experimentation.

This paper provides the first analysis of strategic experimentation in such contexts. It investigates the following questions: Does initial asymmetric information mitigate or exacerbate the free-riding problem? How does it affect agents' experimentation behavior? Does it improve welfare to create information asymmetry in an otherwise symmetric environment?

The central contribution of this paper is to show that initial information asymmetry has a qualitative impact on agents' experimentation behavior: when the information generated from experimentation becomes too discouraging, total experimentation effort can increase, which does not happen without asymmetric information. This is due to the following mutual encouragement effect: a better-informed player's high effort continuously brings good news, encouraging an uninformed player to experiment; in return, the uninformed player's effort pattern raises the reward for high experimentation effort, encouraging the better-informed player to experiment. Moreover, this mutual encouragement effect leads to interesting welfare implications.

To fully explore the impact of initial information asymmetry, this paper builds on the two-player version of the exponential-bandit model (Keller, Rady and Cripps, 2005). At each point in time, each player must divide a unit of resource between a safe project with known payoffs and a risky project of unknown quality. Learning is conclusive: only good risky projects deliver payoffs (breakthroughs), governed by a Poisson process. I add one source of information asymmetry: at date 0, one player, called the informed player (he), privately observes a binary noisy signal, and thus becomes either an optimistic type with a higher posterior than the uninformed player's (she), or a pessimistic type.

With asymmetric information, a public history carries two components of information. One is the information generated from experimentation, depending only on the public history, and thus is called "passive." This component is represented by the informed player's beliefs. Given a public history, how he updates his beliefs is exogenous. The other is the private

[2] The free-riding problem is also well documented empirically; Foster and Rosenzweig (1995) find that during the adoption of high-yielding seed varieties associated with the Green Revolution in India, farmers do not fully incorporate the village returns to learning in making adoption decisions.

information that the informed player leaks to the uninformed player, depending also on the informed player's strategies, and thus is called "strategic." This component is represented by the uninformed player's belief about the informed player being the optimistic type, called his reputation; given a public history, how the informed player's reputation changes depends on the additional information conveyed by his action, which is endogenously determined by his equilibrium strategies.

Before a breakthrough occurs, the passive component of information reduces players' beliefs about the risky project, thereby increasing their incentives to free-ride over time. This component of information is the only one appearing in the symmetric information model (Keller, Rady and Cripps, 2005). With information asymmetry, however, the strategic component comes into play: the informed player maintaining high effort may signal his optimism, thereby bringing good news to encourage the uninformed player to experiment. In turn, the uninformed player's response to his high effort may encourage the informed player to experiment at beliefs he would not if he were alone. As a result, a mutual encouragement effect can arise under asymmetric information, whereas only free-riding is present under symmetric information.

The mutual encouragement effect can qualitatively change the dynamics of players' effort. Specifically, I construct a Markov perfect equilibrium (MPE) using these two components of information as state variables. During a gradual revelation phase of the constructed equilibrium, the pessimistic type mixes between mimicking the optimistic type's high effort and revealing himself. As long as he keeps mimicking, his reputation gradually increases. This rising reputation (induced by the strategic component of information) counterbalances the pessimism induced by the absence of a breakthrough (the passive component), and thus encourages the uninformed player to increase her effort over time.[3]

This rising effort dynamics of the uninformed player occurs when the pessimistic type's belief lies in the region where both players would not experiment if his signal were public and they acted non-cooperatively. Intuitively, during the gradual revelation phase, the pessimistic type has to be indifferent between mimicking the optimistic type's high effort so as to "trick" the uninformed player into exerting effort, and revealing himself, thereby inducing both players to stop experimentation. The marginal value of both players' efforts to the pessimistic type is dropping over time due to the absence of a breakthrough; therefore, for him to be indifferent, the uninformed player's effort has to increase over time. The uninformed player's rising effort can last until the pessimistic type's belief hits the region where experimentation stops, if his signal were public and both players acted cooperatively.[4]

The joint behavior pattern during the gradual revelation phase — the informed player maintaining high effort and the uninformed player increasing her effort despite the absence of a breakthrough — does not occur in any MPE of the symmetric information game,[5] nor

[3]The uninformed player may even become increasingly optimistic about the risky project before a breakthrough occurs, another novel qualitative impact of information asymmetry.

[4]In other words, the pessimistic type plays as if he acts cooperatively with positive probability.

[5]This is due to the strategic substitutability of current effort decisions: the absence of a breakthrough tends to reduce a player's incentive to experiment over time; if the other player does not reduce effort over time, then this player's best response is to reduce effort, using a cutoff strategy.

is it predicted by other papers of the experimentation literature. Moreover, this behavioral pattern admits an intuitive interpretation — leaders motivating followers through role modeling: a leader articulates an appealing vision, which may or may not be reachable; however, as the leader sees further and more accurately than his follower,[6] his putting in long hours during setbacks gradually convinces the follower of his optimism about the vision, and hence motivates the follower to work harder. That leaders enhance followers' commitment to their visions through role modeling is a recurring theme in both modern leadership theories[7] and leadership guidelines in popular management books.[8]

The constructed MPE exists if the initial signal of the informed player is informative enough, and the fraction of the pessimistic type is not too low. The former condition guarantees strict experimentation incentives of the optimistic type during the gradual revelation phase (if he does not worry about his reputation), and the latter guarantees the existence of this phase. If the prior belief is not too low, then the equilibrium path of the constructed MPE is unique among the MPEs such that players play the symmetric MPE after information becomes symmetric and that satisfies a criterion in the spirit of D1.[9]

The mutual encouragement effect leads to interesting welfare implications. Suppose the initial signal is observed by a social planner (she), rather than the informed player. Would she prefer to reveal the signal to both players, or to only one of them?[10] This paper finds that, if the planner cares about ex ante total welfare, and if the binary signal is informative enough, then she prefers the latter. The intuition is as follows. The benefit of asymmetric information, resulting from the mutual encouragement, is enjoyed by both the pessimistic type and the uninformed player, whereas the cost, due to the uninformed player's low effort during the gradual revelation phase, is borne only by the optimistic type. If the initial signal is informative enough, then being still optimistic during the gradual revelation phase, the optimistic type learns little from the uninformed player's experimentation, and therefore suffers little from asymmetric information. As a result, asymmetric information improves (ex ante) total welfare.

Drawing from this welfare implication, a policy maker aiming at promoting new technology adoption may find it desirable to target certain individuals first by giving them relevant information or training. Companies promoting new experience goods might find it profitable to target some consumers, say early adopters, or experts; indeed, pharmaceutical companies spend huge amounts of money targeting marketing activities at "opinion leaders,"

[6] See for instance, page 2 of March and Weil (2009).

[7] For example, in charismatic leadership theory, transformational leadership theory (Bass and Bass, 2009; Yukl, 2010), and in authentic leadership theory (Gardner et al., 2005; Avolio and Gardner, 2005).

[8] For instance, Yukl (2010) gives the following guidelines "for leaders seeking to inspire followers and enhance their self-confidence and commitments to the mission": articulate a clear and appealing vision; explain how the vision can be attained; act confident and optimistic; express confidence in followers; use dramatic, symbolic actions to emphasize key values; and lead by example (role modeling). See page 290–293.

[9] To be precise, the requirement on the informativeness of the informed player's private signal for uniqueness is stronger than the requirement for the existence of the constructed MPE.

[10] Revealing the signal to neither player is always dominated by revealing it to both (in terms of ex-ante total welfare), assuming they play the symmetric MPE afterwards (due to the convexity of the continuation value function associated with this MPE).

for instance, by giving them detailed information about their new drugs, a process called detailing (Nair, Manchanda and Bhatia, 2010).

The joint behavior pattern during the gradual revelation phase is also empirically relevant. It predicts that experienced players experiment more than inexperienced players do and that their experimentation behavior is less sensitive to unfavorable news[11] or other players' experimentation behavior. These predictions are in line with the empirical findings of Bandiera and Rasul (2006), and Conley and Udry (2010). Bandiera and Rasul (2006) study social networks and new crop adoption in Northern Mozambique. They find that experienced farmers are more likely to adopt the new crop than inexperienced ones do and that their adoption decisions are less sensitive to the adoption choices of others. Conley and Udry (2010) investigate learning from one's own and others' experimentation in the diffusion of a new fertilizer in Ghana, and find that a novice farmer's responsiveness to news about the productivity of fertilizer in his information neighborhood is much greater. While such predictions are also compatible with models with myopic players in which experienced players have more precise information, the following prediction drawn from the joint behavior pattern distinguishes the current model from the ones with myopic players: the experimentation behavior of an experienced player with an inexperienced neighbor is less sensitive to bad news than that of an experienced player with a similarly experienced neighbor.

Many empirical papers ignore the strategic component of information that players' actions can convey, even if they allow for information asymmetry. Consequently, when they find players do not adjust experimentation decisions in response to neighbors' news, they reject the existence of learning through experimentation. The joint behavior pattern during the gradual revelation phase implies that such rejection might be incorrect; or even when learning through experimentation is not rejected, ignoring the strategic component of information conveyed by players' actions can lead to underestimating the effect of learning through experimentation.

2 Literature Review

Strategic experimentation with multiple agents was first introduced by Bolton and Harris (1999). In a two-armed Brownian bandit model, they analyze the interaction between two forces — a free-riding effect and an encouragement effect (namely, inter-temporal efforts among teammates are strategic complements), and characterize the unique symmetric MPE. Keller, Rady and Cripps (2005) propose a tractable exponential bandit model to study the strategic experimentation problem, and characterize both the unique symmetric MPE, and other asymmetric MPEs; notably, the encouragement effect is absent in the MPEs of this model.[12] In both papers, players are symmetrically informed. In contrast, this paper finds that, by introducing initial asymmetric information, a new mutual encouragement effect

[11] In this learning from perfect good news model, "no news" is unfavorable news.

[12] The encouragement effect does occur in MPEs with infinite switching. However, an MPE with infinite switching fails to be a limit equilibrium of discrete-time games as the length of a period shrinks to 0.

arises, leading to qualitatively different behavioral and belief dynamics.[13]

This paper is closely related to the recent experimentation literature exploring private learning problems. Bonatti and Hörner (2011) study moral hazard in teams within an exponential bandit setup, in which actions are hidden; they find that players procrastinate (in the unique symmetric equilibrium) and that perfect monitoring on actions exacerbates the procrastination problem even more (in the symmetric MPE). The current paper points out one advantage of perfect monitoring that is absent with symmetrically informed players: signaling by an informed player can push both players to work harder than under symmetric information. Building on Bonatti and Hörner (2011), Guo and Roesler (2016) study a collaboration problem with hidden experimentation actions but with public and irreversible exit decisions. In their model, players may privately learn the quality of their joint project over time if it is bad; as there are payoff externalities, a player who knows the project is bad still stays in the game, delaying the abandonment of socially inefficient projects. Their paper is the closest to my paper in that both papers study signaling in experimentation problems; however signaling plays different roles in the two papers. In my paper, signaling is through experimentation action and thus pushes the informed player to experiment beyond his individual cutoff, whereas in their paper, signaling is through maintaining in the game and the informed player free rides. Therefore, the joint behavior dynamics and welfare implications are qualitatively different.

Private learning is also examined in environment where experimentation decisions are observable but each player may privately learn the quality of risky projects over time. Heidhues, Rady and Strack (2015) analyze a discrete-time version of the exponential bandit model but with private payoffs. They find that if the common prior is sufficiently optimistic, then there is a perfect Bayesian equilibrium that implements the cooperative solution. The basic mechanism is that, if players commit not to reveal the breakthrough, then learning slows down, and hence belief deteriorates sluggishly in case a player hasn't experienced a breakthrough; when the common prior is sufficiently optimistic, the players are indeed able to commit, because if so, at the cooperative cutoff, players observing no breakthrough are still more optimistic than the myopic cutoff belief and hence willing to experiment. A similar mechanism is the driving force of Das (2015).[14] In contrast, the mechanism in this paper is through signaling, which can sustain experimentation even when the informed player's belief lies in the region where he is too pessimistic to experiment alone. The dynamics of behavior and belief in their papers are qualitatively different from that in mine: in their papers, players' effort paths (on the equilibrium path) are of cutoff types; neither effort nor belief can increase over time before they learn the quality of the risky project.

More broadly, that asymmetric information may improve total welfare also relates to the

[13]In Bolton and Harris (1999), higher future efforts by other players increase a player's *continuation value*, thereby encouraging the player to experiment to collect the continuation value. In my paper, a higher reputation of the informed player increases both the uninformed player's instantaneous payoff and her continuation value, thereby encouraging her to experiment.

[14] Das (2015) studies a competitive environment where efforts and payoffs are public but players may privately learns the type of the risky project over time if it is good; only the player who experiences the first public breakthrough receives a reward.

leadership literature. Hermalin (1998) and Komai, Stegeman and Hermalin (2007) analyze a static model of moral hazard in team, in which the leader (who knows the state of the world) signals to the followers the value of their joint project by working hard, thereby partially overcoming the free-riding problem. Different from them, this paper focuses on a dynamic model, aiming at explaining the dynamic provision of (informational) public goods, which cannot be analyzed in their static setup. Moreover, the welfare implications are different: when the informed player knows the risky project is good but does not know it is bad, creating information asymmetry improves welfare in my setup (with purely informational externalities), whereas it is not always so in their setup (with purely payoff externalities).

The property that the uninformed player increases her effort over time despite her deteriorating belief resembles herding behavior studied by the herding literature (for instance, Banerjee, 1992; Scharfstein and Stein, 1990), in the sense that the uninformed player herds with the informed player. Different from the herding literature, the uninformed player in this paper does not ignore her information; in fact, she uses all her information. The reason she "herds" with the informed player is because the informed player's action conveys enough encouraging information to offset the discouraging public information.

Finally, the fact that the uninformed player benefits from her ignorance in equilibrium bears a similarity with the advantage of using arm's length relationships by a principal who cannot commit in the contracting literature (Crémer, 1995), and the self-discipline function of strategic ignorance to a time-inconsistent decision maker in the self control literature (Carrillo and Mariotti, 2000). Crémer (1995) finds that, a principal unable to commit not to renegotiate a contract is better off with an arm's length relationship because it gives her commitment power to punish an agent's poor performance. Carrillo and Mariotti (2000) find that a time-inconsistent decision maker may forgo free useful information, in fear that her future selves would not be able to commit to the optimal consumption plan she makes today, after observing the information. In the current paper, the uninformed player would turn down the opportunity to learn the informed player's information freely, knowing that she would not commit to encouraging the informed player if she knows he is the pessimistic type, dampening the experimentation incentives of the latter.

3 The Model

Time is continuous, indexed by $t \in [0, \infty)$. There are two players. Each player is endowed with one unit of a divisible resource per unit of time, and divides it between a safe project and a risky project. A safe project delivers a known return; the return of a risky project depends on its quality θ, unknown and common to both players, with $\theta = g$ referring to a good project, and $\theta = b$ to a bad one. Over any time interval $[t, t + dt]$, if a player allocates $e_t \in [0, 1]$ resource to the risky project, and hence $(1 - e_t)$ to the safe project, he or she receives $(1 - e_t)sdt$ from the safe project, and a lump-sum payoff h with probability $e_t \lambda \mathbb{1}_{\{\theta=g\}} dt$ from the risky project, with $\lambda > 0$. That is, a bad risky project delivers nothing whereas a good risky project delivers Poisson payoffs, implying that one single arrival of the lump-sum payoff perfectly reveals to a player his or her risky project being good; a lump-sum payoff is thus called a breakthrough. At any time t, players observe all past experimentation decisions and experimentation outcomes.[15] Both players prefer a good risky project to a safe project, and a safe project to a bad risky project: $\lambda h > s > 0$. They discount future payoffs with a common discount rate $r > 0$.

Initially, players share a common prior q_0, the probability with which each player believes his or her risky project being good. At time 0, one player, called the informed player (player I, he), receives signal s_- with probability ρ_θ, and s_+ with probability $1-\rho_\theta$. Signal s_- is more likely to occur to a bad risky project than to a good risky project: $1 \geq \rho_b > \rho_g > 0$. By Bayes' rule, after receiving signal s_-, I adjusts his belief downward to some q_0^-, strictly lower than the uninformed player's (player U, she) posterior q_0, thereby becoming a pessimistic type; otherwise, he adjusts his belief upward to some $q_0^+ > q_0$, thereby becoming an optimistic type. The parameters q_0, ρ_b, and ρ_g are common knowledge.[16] The initial information asymmetry is the only divergence from the two-player version of the canonical exponential-bandit model (Keller, Rady and Cripps, 2005)

Note that as in the canonical multi-player exponential-bandit model, there is only informational externality, because each player obtains payoffs only from his or her own projects, independent of the other player's actions conditional on the quality of the projects.

Remark. [A joint project interpretation] Because learning is conclusive and the experimentation decisions and outcomes are public, experimentation ends once a breakthrough occurs. And because there is only informational externality, the game essentially ends after the arrival of the breakthrough; it then becomes a dominant strategy for a player to use the risky project forever, bringing a discounted payoff $\lambda h/r$. Note also that the player who receives the first breakthrough enjoys an additional payoff h relative to the other. Therefore, the model admits the following joint project interpretation: instead of working on two risky projects of the same quality, the two players work on one joint risky project; a breakthrough occurs to the risky project with the same probability as in our model, bringing a lump-sum payoff $\lambda h/r$ to each, and an additional intrinsic satisfaction h to the first player who experiences the breakthrough; the project is completed after the breakthrough, and hence the game ends.

3.1 The cooperative solution

If players act cooperatively to maximize their joint surplus, the informed player would reveal his signal truthfully to the uninformed player. Therefore, from time 0 on, both players would share a common posterior q_t.[17] This posterior q_t continuously decreases over time as long as players experiment and a breakthrough does not arrive. Both players then adopt a

[15] Specifically, if we use (e_s^I, e_s^U) to denote I's and U's efforts taken at time s, then at time t before players' actions, both players can observe the effort history before t, $(e_s^I, e_s^U)_{s<t}$, and the experimentation outcome history before t.

[16] Such initial information asymmetry arises for instance if I is an incumbent, U an entrant of the experimentation game, if U is not sure exactly how much I has experimented before time 0.

[17] Message-sending is redundant here; the informed player's stopping time is enough to communicate his private information.

cutoff strategy: experimenting if q_t is higher than a cutoff $q_2^* \in (0, 1)$, defined by

$$r(\lambda q_2^* h - s) + 2\lambda q_2^*(\lambda h - s) = 0, \tag{1}$$

and stopping otherwise. To understand equation (1), note that the first term on the left-hand side is the flow marginal benefit of experimentation at the cooperative cutoff q_2^*, and the second term the continuation marginal benefit to the two players. Equation (1) says at the optimum, the total marginal benefit of experimentation is 0 (a smooth pasting condition).

For future use, we also introduce the individually optimal solution, a similar cutoff strategy with cutoff q_1^*, where q_1^* satisfies a similar equation with equation (1) except that the index 2 is replaced by 1. Since the continuation benefit to two players is twice as much as that to a single player (at the same belief) whereas the flow net benefit is the same, we have $q_2^* < q_1^*$. That is, two players experimenting cooperatively acquire more information than does a single player.[18]

4 Beliefs and Equilibrium Concept

Following experimentation papers with symmetric information (for instance, Bolton and Harris, 1999; Keller, Rady and Cripps, 2005), we focus on Markov perfect equilibrium (MPE). However, different from them, there is no single state variable that can be used for the solution concept, because players do not share a common posterior. Observe that a public history carries two components of information. The first is the information obtained from the experimentation technology, depending only on the public history, independent of players' equilibrium strategies, hence is called "passive." This component of information can be represented by how the informed player updates his beliefs. The other is the informed player's private information leaked into public through his actions, depending also on his equilibrium strategies and hence is called "strategic." This component of information can be represented by how the uninformed player updates her belief about the informed player being an optimistic type. This strategic component is absent from a symmetric information setup, and can potentially affect U's belief about the risky project in the opposite direction that the passive component does, and hence cannot be represented by the passive component. Based on this observation, we define state variables. Strategies, belief system, and equilibrium are defined afterward.[19]

4.1 The state variables

The passive component—the background belief. Consider a naive outsider (he) who knows the model setup except that he mistakenly thinks neither player observes the

[18]Following the terminology of Keller, Rady and Cripps (2005), the *amount* of information acquired is measured by the probability of learning the true state given the prior, whereas the *intensity* of experimentation refers to the efforts of players.

[19]As we will see in Section 4.2, the belief system is also assumed to be Markov.

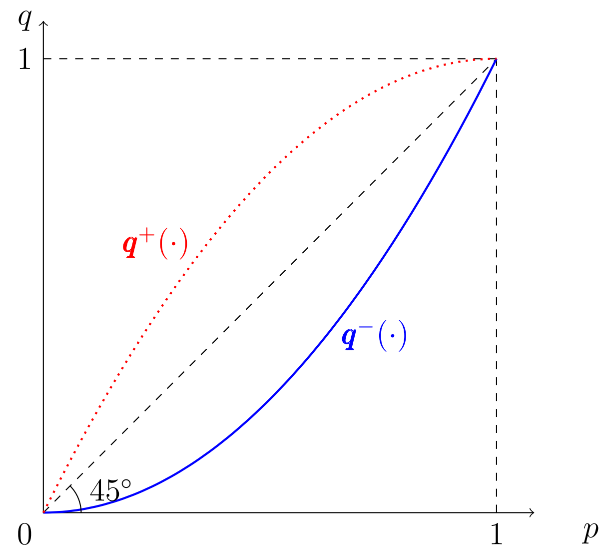

Figure 1: True posteriors and the background belief

initial binary signal (of I's); that is, he mistakenly thinks players have symmetric information. Assume he starts with the same prior $p_0 \equiv q_0$ and observes the same public histories as our players do. Denote his posterior as p, and call it the *background belief*.[20]

Of course, this background belief differs from I's posterior. But if after a public history, the naive outsider is told of I's private signal, he then would adjust his belief to exactly I's. That is, after any public history, if the background belief is p, type s_-'s posterior must be $\boldsymbol{q}^-(p)$, given by Bayes rule,

$$\boldsymbol{q}^-(p) = \frac{p\rho_g}{p\rho_g + (1-p)\rho_b}, \tag{2}$$

and type s_+'s must be $\boldsymbol{q}^+(p)$, given by

$$\boldsymbol{q}^+(p) = \frac{p(1-\rho_g)}{p(1-\rho_g) + (1-p)(1-\rho_b)}. \tag{3}$$

Figure 1 illustrates the relationships between the two posteriors $\boldsymbol{q}^-(p)$, $\boldsymbol{q}^+(p)$, and the background belief p, with the dotted line referring to $\boldsymbol{q}^+$ and the solid line to $\boldsymbol{q}^-$.

Equations (2) and (3) imply that the background belief p and the signals of the informed player, s_- and s_+, are sufficient to track the informed player's posteriors. To track U's belief about the risky project, we still need the strategic component of information.

The strategic component—I's reputation: the probability U assigns to I being type s_+, denoted by μ. Together with the background belief, I's reputation determines U's posterior about the risky project by

$$\boldsymbol{q}^U(p,\mu) \equiv \mu\boldsymbol{q}^+(p) + (1-\mu)\boldsymbol{q}^-(p), \tag{4}$$

[20] A formal definition of the background belief is given in Appendix A.

and hence directly affects U's flow experimentation payoff. As U's experimentation incentives are affected by I's effort strategies, which depend on I's types, the weight that U puts on each type is necessary to compute U's continuation experimentation payoffs. It is through this second way that I's reputation is indispensable for the equilibrium analysis.[21] I does not care about his reputation per se, but because U cares about it and will choose her strategy accordingly, I cares about it indirectly.

In sum, the background belief, p, and I's reputation, μ, are sufficient to represent the two components of information and are thus used as state variables.

To reduce the burden of notation, denote the expected arrival rate (of breakthrough under full effort) for type s_+, type s_-, and U at state (p, μ), as $\lambda^{I+}(p)$, $\lambda^{I-}(p)$, and $\lambda^{U}(p, \mu)$, respectively, which are their posterior beliefs about the risky project multiplied by the arrival rate of breakthroughs of a good risky project λ.

Remark. Singling out this non-strategic background belief from a public history is not only technically convenient, but also empirically relevant. We interpret the outsider that we introduce to define the background belief as an econometrician who mistakenly thinks players are symmetrically informed. Therefore, the true asymmetric information model will be misspecified as a symmetric information model by this econometrician. We will discuss the empirical consequences of such a misspecification later.

4.2 Strategies and belief system

Players' strategies are Markov in the state variables (p, μ). A pure strategy for U, type s_+, and type s_- are denoted by e^U, e^{I+}, and e^{I-} respectively, with $e^U(p, \mu)$ referring to U's effort level at state (p, μ). We are interested in equilibria in which both U and type s_+ play pure strategies, and type s_- plays a pure strategy after his type is (truthfully) revealed. In such MPEs, a mixed strategy of type s_- is a mixture over his pure strategies, and can be defined based on Aumann (1964).

The belief system is denoted by $\mu(s_+|\cdot)$, with $\mu(s_+|e^I, p, \mu)$ being the probability that U assigns to I being type s_+ in state (p, μ), once she observes that I exerted effort e^I. Here we adapt the belief system to be Markov to suit our definition of MPE.

4.3 Equilibrium

Given a Markov strategy profile (e^{I-}, e^{I+}, e^U) and a belief system $\mu(s_+|\cdot)$, the expected average payoff to type s_l, $l \in \{+, -\}$, at time 0 is

$$E\left[\int_0^\infty re^{-rt}\left(\left(1-e_t^{Il}\right)s + e_t^{Il}\lambda h\theta\right)dt \mid e^{I-}, e^{I+}, e^U, \mu\left(s_+|\cdot\right)\right],$$

[21] It is true that for a fixed background belief p, equation (4) defines a one-to-one mapping between q^U and μ; hence equivalently, we can use (p, q^U) as the state variables. However, this equivalence will not hold if I has more than two types; q^U will not be sufficient to calculate the weight that U assigns on each type. In such circumstances, the relevant state variables will be the background belief, and the probability that U assigns to each type (that is, her belief about I's belief).

which is equal to

$$E\left[\int_0^\infty re^{-rt}\left(\left(1-e_t^{Il}\right)s+e_t^{Il}\lambda^{Il}(p_t)h\right)dt \mid e^{I-},e^{I+},e^U,\mu\left(s_+|\cdot\right)\right], \tag{5}$$

by the Law of Iterated Expectations.

Similarly, the expected average payoff of player U at time 0 is

$$E\left[\int_0^\infty re^{-rt}\left(\left(1-e_t^{U}\right)s+e_t^{U}\lambda^{U}(p_t,\mu_t)h\right)dt \mid e^{I-},e^{I+},e^U,\mu\left(s_+|\cdot\right)\right].$$

A strategy profile (e^{I-},e^{I+},e^U) and a belief system $\mu\left(s_+|\cdot\right)$ is an MPE if given the other player's strategy and the belief system, a player finds it optimal to play her equilibrium strategy, and if the belief system is consistent.

4.4 The evolution of the state variables

Given an action path $\{(e_t^I,e_t^U)\}_{t\geq 0}$ (on or off the equilibrium path), before a breakthrough occurs, the background belief process $\{p_t\}_{t\geq 0}$ evolves according to

$$dp_t=-p_t(1-p_t)(e_t^I+e_t^U)\lambda dt, \tag{6}$$

by Bayes' rule.[22]

The evolution of the reputation processes depends on equilibrium prescription. Fix an equilibrium $(e^{I+},e^{I-},e^U;\mu(s_+|\cdot))$. We focus on the histories along which *no breakthrough has occurred and I has not revealed his type* (on the equilibrium path).[23]

If the equilibrium is full pooling over $[0,T]$ for some T, then U's belief about the risky project coincides with the background belief over $[0,T]$; as a result, I's reputation at background belief p_t for any $t\in(0,T]$ is equal to

$$\boldsymbol{\mu}^o(p_t)\equiv p_t(1-\rho_g)+(1-p_t)(1-\rho_b), \tag{7}$$

where $\boldsymbol{\mu}^o$ is called a *full pooling path*. Since $p_0=q_0$, $\boldsymbol{\mu}^o\left(q_0\right)$ refers to I's reputation at time 0. As p_t decreases over time, so does I's reputation during full pooling; intuitively, since signal s_+ is more likely to occur to a good risky project, as U becomes more pessimistic about the risky project being good, so does she about s_+ having occurred.

[22] To see this, suppose at posterior p_t, players take efforts (e_t^I,e_t^U) during a dt duration of time. If a breakthrough does not arrive during this interval, the players' posterior at $t+dt$, p_{t+dt}, is

$$p_{t+dt}=\frac{p_t(1-(e_t^I+e_t^U)\lambda dt)}{p_t(1-(e_t^I+e_t^U)\lambda dt)+(1-p_t)}$$

by Bayes rule. Using this equation, the belief change in this interval conditional on no breakthrough having occurred, $dp_t\equiv p_{t+dt}-p_t$, is given by equation (6).

[23] Once I's type is revealed, then μ either stays at 0 or 1 on the equilibrium path; once a breakthrough occurs, I's reputation ceases to matter.

Once the equilibrium diverges from full pooling, I's reputation μ_t would differ from $\boldsymbol{\mu}^o(p_t)$. Specifically, the equilibrium $(e^{I+}, e^{I-}, e^U; \mu(s_+|\cdot))$ induces a distribution over public histories, which defines a cumulative distribution function (CDF) over type s_-'s stopping times (of mimicking type s_+) along the outcome path such that *no breakthrough has occurred and I has not revealed his type*. Denote this CDF as Y, with Y_t referring to the cumulative probability that type s_- has revealed himself before and at time t.[24]

By Bayes' rule, I's reputation μ_t satisfies

$$\mu_t = \frac{\boldsymbol{\mu}^o(p_t)}{\boldsymbol{\mu}^o(p_t) + [1 - \boldsymbol{\mu}^o(p_t)]\,(1 - Y_t)}. \tag{8}$$

Written in its differential form, the reputation process $\{\mu_t\}_{t\geq 0}$ evolves according to

$$\frac{d\mu_t}{\mu_t(1-\mu_t)} = \frac{d\boldsymbol{\mu}^o(p_t)}{\boldsymbol{\mu}^o(p_t)\,(1 - \boldsymbol{\mu}^o\,(p_t))} + \frac{dY_t}{1 - Y_t}, \tag{9}$$

where $\frac{dY_t}{1-Y_t}$ denotes the probability that type s_- reveals himself over dt interval of time, conditional on *no breakthrough having occurred and I having not revealed his type.*

4.5 The continuation game under symmetric information

This paper focuses on the MPEs such that in the continuation game after I's type is revealed,[25] players play the unique symmetric MPE, denoted as $w^S(\cdot)$ with their common posterior as the state variable. In this MPE, as characterized by Keller, Rady and Cripps (2005), each player puts all the resource in the risky project when optimistic (when the common posterior is in $[q^S, 1]$), all the resource in the safe project when pessimistic (when the common posterior is in $[0, q_1^*]$), and an interior level of resource in both projects otherwise. That is, in this MPE, players stop experimentation when their common posterior is below the individual cutoff q_1^*.

Due to the one-to-one relationships between players' true posteriors and the background belief, given in equations (2) and (3), the continuation equilibrium can be equivalently expressed using the background belief as the state variable. Figure 2 gives an illustration. In the continuation equilibrium after type s_-'s signal becomes public, illustrated by the dashed

[24] To be precise, a strategy profile $\boldsymbol{e} \equiv (e^{I+}, e^{I-}, e^U)$ induces a probability distribution $\mathbb{P}_{p_0,\boldsymbol{e}}$ over public histories. Let N_t denote the number of breakthroughs that have occurred until time t. Let H_t^r denote the set of time-t public histories along which I stops taking type s_+'s prescribed action before a breakthrough occurs, that is, $H_t^r \equiv \{(e_s^I, e_s^U, N_s)_{s<t} : \inf\{s : e_s^I \neq e_s^{I+}\} \leq \inf\{s : N_s \neq 0\}\}$. Let h_t^n denote the public history that I has been taking type s_+'s prescribed action and that a breakthrough has not occurred till t (excluding t). Then the cumulative probability of separation at time t before I's move, Y_{t-}, is

$$\mathbb{P}_{p_0,\boldsymbol{e}}(H_r^t | H_r^t \cup \{h_n^t\}).$$

[25] "After I's type is revealed" means after I's type is truthfully revealed, that is, once information between the two players is symmetric.

curve in Figure 2, each player puts all the resource in the risky project when the background belief p is greater than p^{S-}, defined by $q^S = \boldsymbol{q}^-\left(p^{S-}\right)$, and all the resource in the safe project when p is smaller than p_1^{*-}, defined by $q_1^* = \boldsymbol{q}^-\left(p_1^{*-}\right)$.

Similarly, the solid curve in Figure 2 illustrates the continuation equilibrium after type s_+'s signal becomes public, with p^{S+} and p_1^{*+} defined likewise.

To avoid redundancy, whenever no confusion arises, we call p_1^{*-} type s_-'s individual cutoff (background belief), p_2^{*-} his cooperative cutoff, and p^{S-} his switching cutoff. Type s_+'s cutoffs, p_1^{*+}, p_2^{*+}, and p^{S+}, are similarly defined.

4.6 The odds ratio

A crucial factor driving the momentum of the mutual encouragement effect is the belief gap between I's two types. It is naturally measured by an *odds ratio* $a \equiv \frac{(1-\rho_g)/\rho_g}{(1-\rho_b)/\rho_b}$, the ratio of the odds of signal s_+ occurring to a good risky project to the odds of it occurring to a bad risky project. This is because, from equations (2) and (3), the odds ratio is also equal to

$$a = \frac{\boldsymbol{q}^+(p)}{1-\boldsymbol{q}^+(p)} \Big/ \frac{\boldsymbol{q}^-(p)}{1-\boldsymbol{q}^-(p)}, \tag{10}$$

the ratio of the odds that type s_+'s risky project is good to the odds that type s_-'s risky project is good. In this sense, it measures both the informativeness of I's private signal, and also the belief gap between I's two types.

The following assumption greatly eases our exposition of the mutual encouragement effect. Section 6 discusses what happens if this assumption does not hold.

Assumption 1. *The odds ratio a is greater than or equal to $a^S \equiv \frac{q^S}{1-q^S} / \frac{q_2^*}{1-q_2^*}$.*

Under Assumption 1, when type s_-'s belief is equal to the cooperative cutoff q_2^*, type s_+'s would be greater than or equal to the switching cutoff q^S (at the same public history). It means the belief gap between I's two types differs sufficiently, in such a way that after the players with public information s_- find it optimal to stop experimenting when playing cooperatively, the players with public information s_+ still experiment with full resource for at least some time (when playing the symmetric MPE). The MPEs in Figure 2 satisfy this assumption, because in this figure, $p_2^{*-} > p^{S+}$, which is equivalent with $\boldsymbol{q}^+\left(p_2^{*-}\right) > \boldsymbol{q}^+\left(p^{S+}\right)$, meaning that type s_+'s posterior when type s_-'s is at q_2^* (that is, $\boldsymbol{q}^+\left(p_2^{*-}\right)$), is greater than q^S.

5 MPE with Gradual Revelation

We now construct the MPE of interest. We first highlight the main features of the mutual encouragement effect, and then elaborate its implications for the equilibrium behavior dynamics and belief dynamics. Detailed construction of the equilibrium is postponed to the

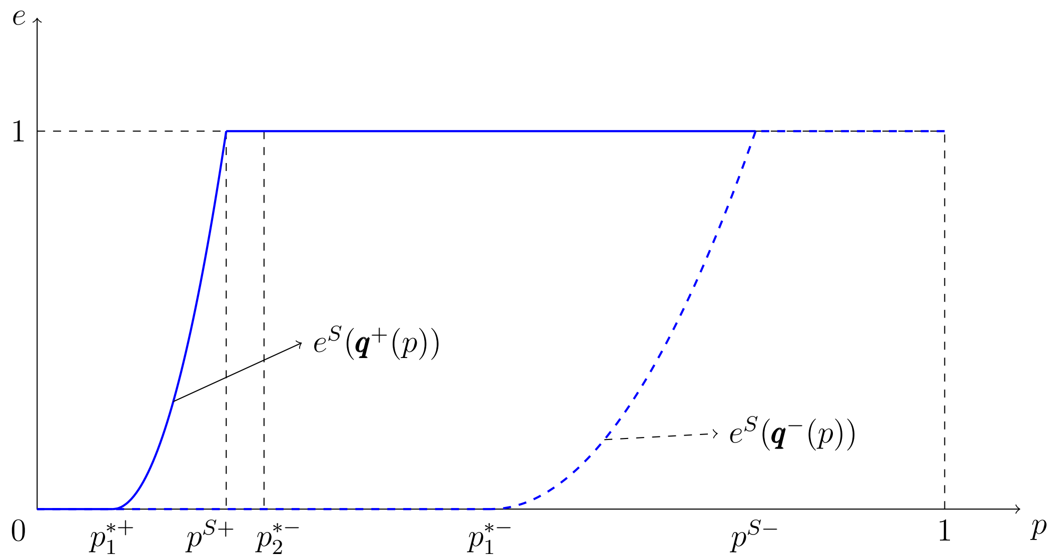

Figure 2: The symmetric MPE as functions of the background belief

final subsection. Section 6 discusses the issue of equilibrium multiplicity. All equilibrium descriptions are conditioned on the histories such that no breakthrough has occurred.

In the MPE of interest, after I's type is revealed, the players play the symmetric MPE; therefore, whenever we say type s_- reveals himself, we mean that he plays this MPE strategy, and immediately after this, U follows suit. This MPE has three phases.

1. When type s_- is sufficiently optimistic — over background beliefs $(p_{gr}, 1)$, p_{gr} to be determined — the equilibrium involves *full pooling*, during which, both players allocate all resources to the risky projects; as a result, I's reputation gradually decreases over time (along the full pooling path $\boldsymbol{\mu}^o(p_t)$). The eroding reputation path is illustrated by the dash-dot line over the interval $(p_{gr}, 1]$ in Figure 3; as time passes by, I's reputation descends along this line from right to left until p reaches p_{gr}.

   When the prior q_0 is greater than p_{gr}, this phase happens at the beginning of the game; otherwise, the equilibrium path does not involve this phase.

2. When type s_-'s belief is intermediate — over background beliefs (p_2^{*-}, p_{gr}) — the equilibrium involves *gradual revelation*, during which, type s_+ still allocates all resources to the risky project, whereas type s_- mixes between mimicking type s_+ and revealing himself, such that as long as he keeps mimicking, his reputation gradually rises, along a "gradual revelation path" (function) $\hat{\boldsymbol{\mu}}(p_t)$. This rising reputation path is illustrated by the solid curve in Figure 3; as time passes by, I's reputation ascends along this line from right to left.

   If the prior q_0 is greater than p_{gr}, this phase happens immediately after the full pooling phase; if q_0 is between p_2^{*-} and p_{gr}, this phase happens at the beginning of the game; otherwise, the equilibrium path does not involve this phase.

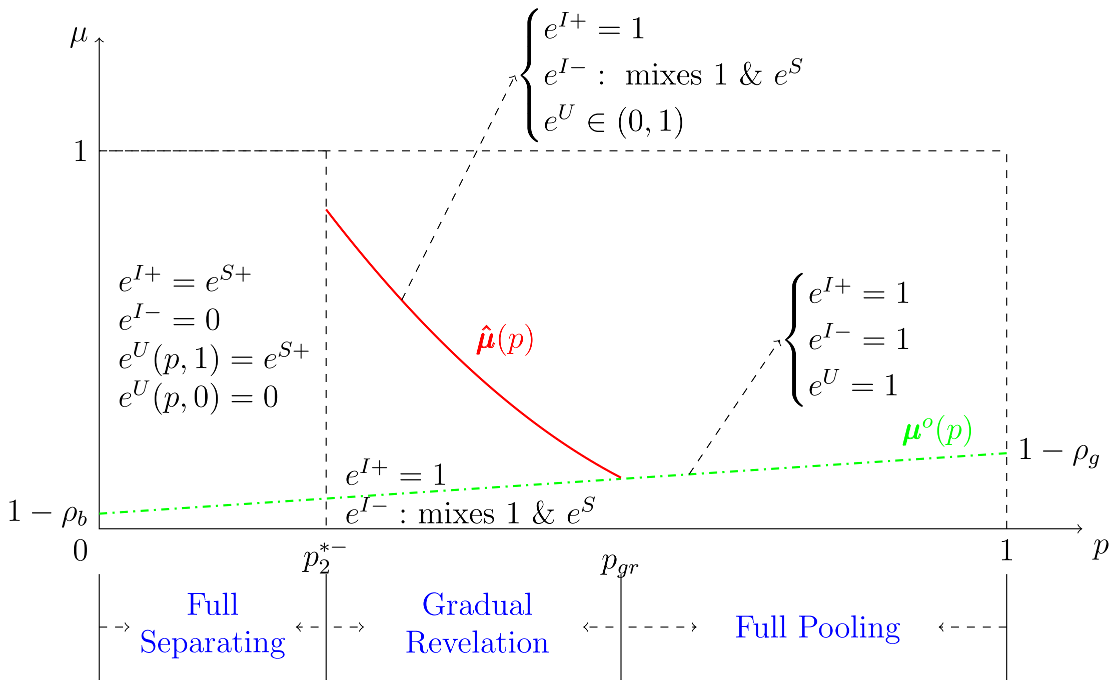

Figure 3: An MPE with gradual revelation

3. When type s_- is sufficiently pessimistic — over background beliefs $(0, p_2^{*-})$ — the equilibrium involves *full Separation*, during which, type s_+ plays the symmetric MPE strategy under symmetric information s_+, whereas type s_- stops experimenting immediately. Referring to Figure 3 again, if I stops experimentation, the state variables jump on the line $\mu = 0$ and then freeze there; otherwise, the state variables jump on the line $\mu = 1$ and move along it from right to left until experimentation stops.[26]

   If the prior q_0 is greater than p_2^{*-}, this phase happens immediately after the gradual revelation phase; otherwise, the equilibrium path only involves this phase.

Call this equilibrium an "MPE with gradual revelation". Figure 4 depicts two typical evolution paths of the state variables in this MPE, conditional on I being type s_+ and a breakthrough having not arrived. When the prior q_0 lies in the full pooling region $(p_{gr}, 1)$, say on the closed circle of $\boldsymbol{\mu}^o$, as time goes by, the state variables move from right to left, along $\boldsymbol{\mu}^o$, $\hat{\boldsymbol{\mu}}$, and finally $\mu = 1$, as represented by the solid arrowed curve. When the prior lies in the gradual revelation region (p_2^{*-}, p_{gr}), for instance at the open circle of $\boldsymbol{\mu}^o$, type s_- reveals with a positive probability such that upon non-revealing, the state variables immediately jump up on the curve $\hat{\boldsymbol{\mu}}$, and move in the same way as along the dotted arrowed line afterward.

Proposition 1 presents the main result of this section — the qualitative features of the behavior dynamics and belief dynamics, with the quantitative ones postponed till the last subsection.

[26] Of course, the state variables freeze up when the background belief reaches p_1^{*+}, below which, even type s_+ stops experimenting. We do not give a separate name to the equilibrium over $[0, p_1^{*+}]$ because it is trivial.

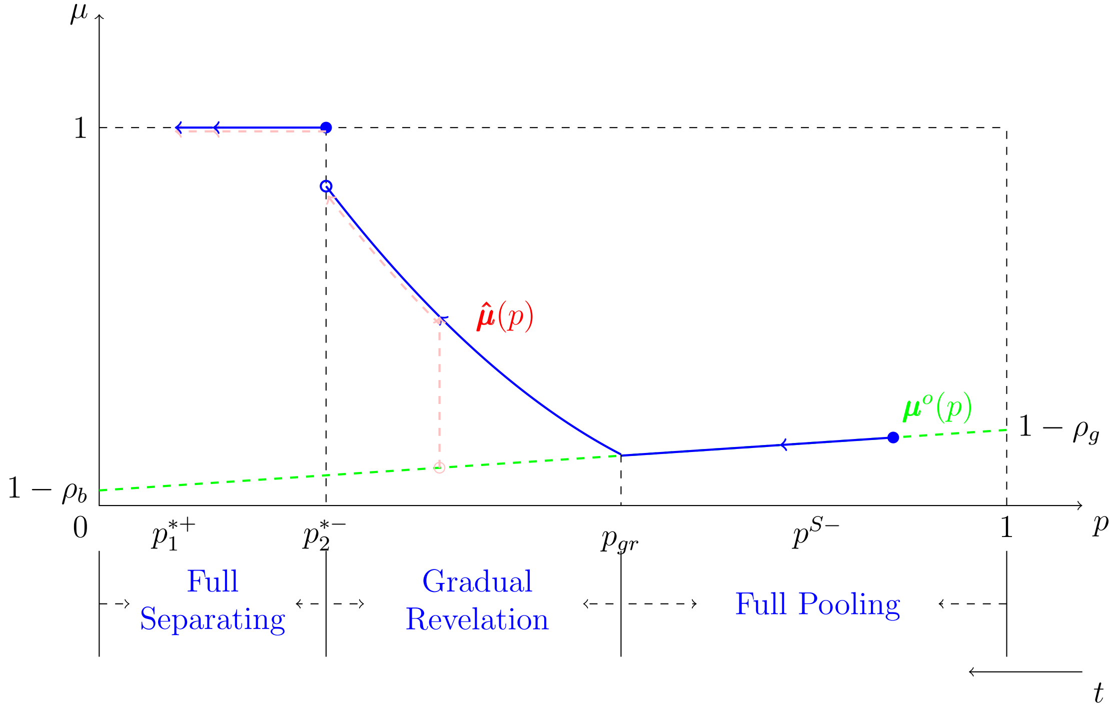

Figure 4: An MPE with gradual revelation: two paths of the state variables

Note: both paths are conditioned on no breakthrough having occurred and the informed player being type s_+. The (blue) solid arrowed curve represents the path corresponding to a high prior q_0 (lying in the full pooling phase); the (pink) dashed arrowed curve represents the path corresponding to a intermediate prior q_0 (lying in the gradual revelation phase).

Proposition 1. *If the MPE with gradual revelation exists, then during the gradual revelation phase, as long as a breakthrough has not arrived and type s_- has not revealed himself, over time,*

1. *U's effort gradually increases, when the background belief is between type s_-'s cooperative cutoff p_2^{*-} and his single-player cutoff p_1^{*-};*
2. *I's reputation gradually rises;*
3. *if the informativeness of I's initial signal is intermediate (that is, if the odds ratio a is not too high but still satisfies Assumption 1), U's belief about the risky project is either increasing or U-shaped.*

The rising reputation is already presented in Figure 4; we now illustrate the other two results. Figure 5 displays the uninformed player's effort path corresponding to the blue reputation path in Figure 4, by an arrowed curve. As in Proposition 1, U increases her effort over time when the background belief is between p_1^{*-} and p_2^{*-}. We will discuss U's decreasing effort part (over time) during the gradual revelation phase in Lemma 2 of Section 5.4.1; U's decreasing effort part (over time) during the full separation phase coincides with that in the symmetric MPE under public information s_+ (the solid curve in Figure 2).

Figure 6 and 7 contrast two distinct paths of U's belief about the risky project; in both figures, the horizontal axis represents type s_-'s posterior, the dotted curve type s_+'s, and the solid curve the uninformed player's. In Figure 6, which corresponds to a large odds ratio, U's belief decreases over time before the full separation phase occurs, as under symmetric information; but in Figure 7, which corresponds to an intermediate odds ratio, and hence to the third result of Proposition 1, U's belief is U-shaped before the full separation phase occurs.

Therefore, compared with the symmetric MPE under symmetric information, in which players' efforts and beliefs monotonically decrease over time, and they stop experimentation at the single-player cutoff , two features of the current MPE stand in sharp contrast.

First, the uninformed player can *increase effort*, and *become more optimistic* about the risky project over time, despite the deterioration of the background belief. She does so because I's high effort continually brings in good news, compensating for the absence of a breakthrough, and encouraging her to experiment. Here thus lies the first layer of the mutual encouragement effect.

Second, the pessimistic type *experiments beyond the single-player cutoff*, all the way till the cooperative cutoff, with positive probability. He does so because the uninformed player responds to his hard work by also working hard, indirectly inducing the pessimistic type to internalize U's benefit from his effort, encouraging him to experiment at beliefs he would not if he were alone or his signal were public. Thus, the first layer of the mutual encouragement effect brings forth a second layer of the effect.

We therefore have identified the following mutual encouragement effect: I's rising reputation compensates the dropping background belief, encouraging U to experiment; U's

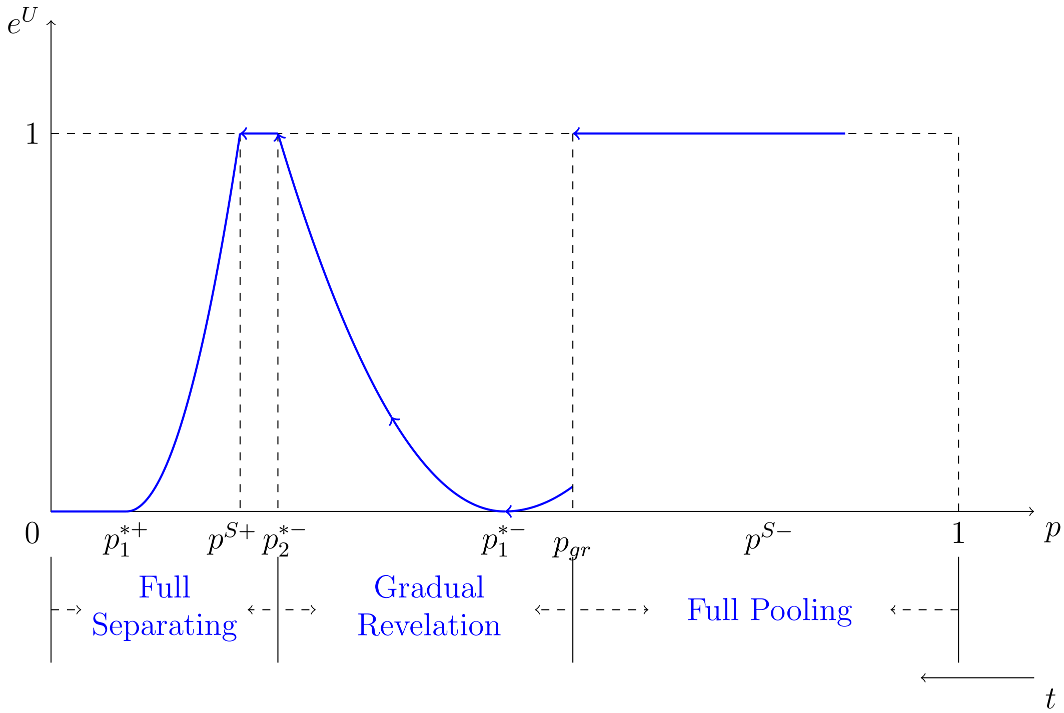

Figure 5: The uninformed player's effort

Note: U's effort path is conditioned on no breakthrough having occurred and I being type s_+. It increases over time when p is between p_1^{*-} and p_2^{*-}, that is, when type s_-'s belief is between the single-player cutoff and the cooperative cutoff.

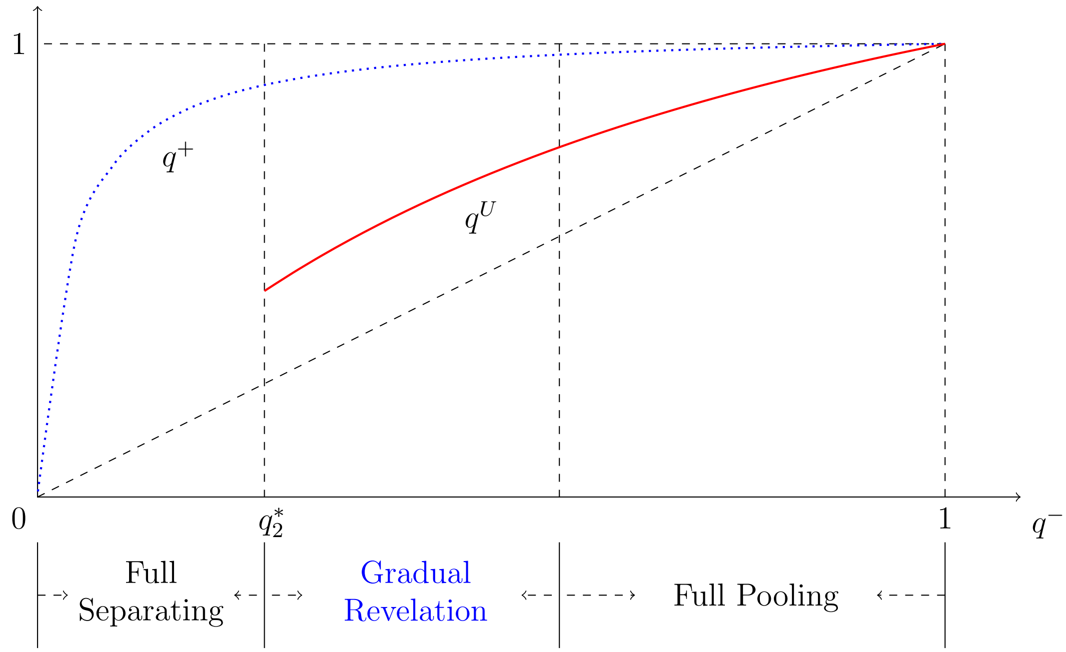

Figure 6: U's growing pessimism before full separation (a large odds ratio)

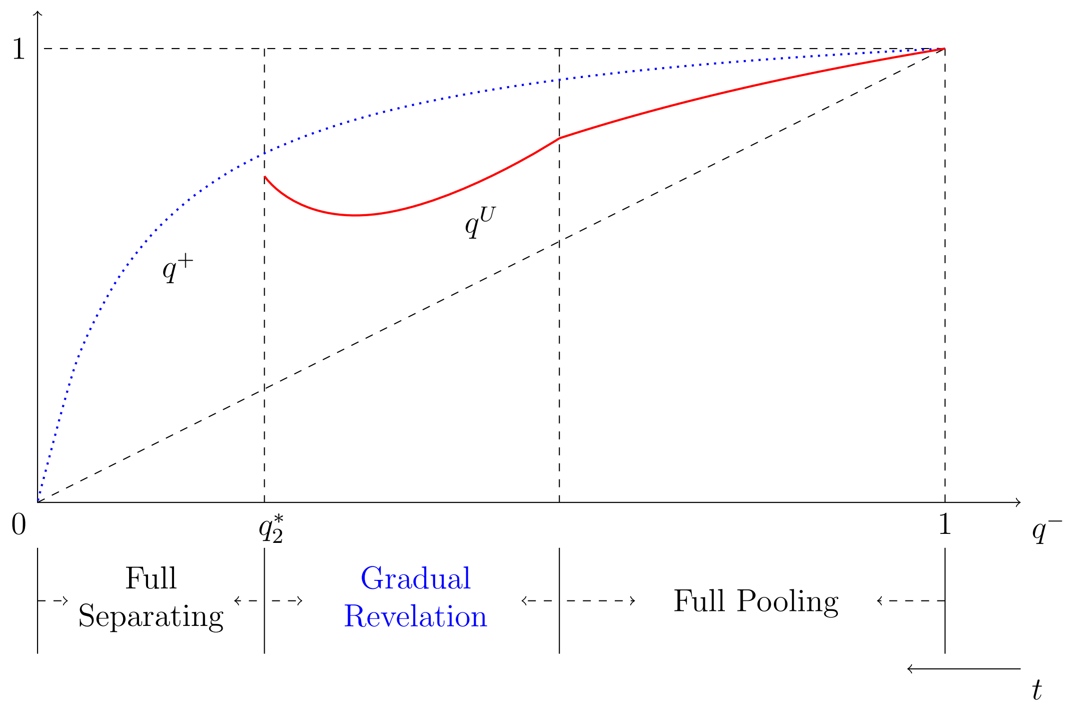

Figure 7: U's growing optimism before full separation (an intermediate odds ratio)
Note: when the odds ratio is large (Figure 6), U's belief about the risky project q^U is decreasing over time as long as no breakthrough has occurred and I has not revealed himself. Whereas when the odds ratio is intermediate (Figure 7), U's belief q^U is first decreasing over time and then increasing; in particular, it increases right before the full separation phase occurs.

increasing effort compensates the growing pessimism of type s_-, encouraging him to experiment (till the cooperative cutoff with positive probability). Driven by this effect, the joint behavior pattern — the informed player keeps exerting high effort while the uninformed player increases effort, despite the absence of a breakthrough — does not occur in any MPE that is a limit MPE of the symmetric information discrete-time games, not just in the symmetric MPE.[27] This pattern leads to qualitatively different empirical predictions, as we will discuss in Section 8.

Remark (Divergent learning dynamics.)**.** Note that if the gradual revelation phase starts at a background belief higher than type s_-'s individual cutoff (that is, $p_{gr} > p_1^{*-}$),[28] then with positive probability, type s_- reveals himself at background beliefs higher than p_1^{*-}. After this, both players play the symmetric MPE (see the dashed curve in Figure 2), meaning that both players exert little effort as p moves close to p_1^{*-}, and hence learning is slow. On the other hand, with positive probability, type s_- continues experimenting with high effort before experimentation stops; we interpret his behavior as a leader "leading by example" (that is, acting as a role model, as discussed in the introduction).

Therefore, combining the joint project interpretation of our model (page 8), the MPE of interest predicts that two identical groups working on the same joint projects and receiving the same information can exhibit different learning dynamics. In one group in which the informed player leads by example, learning is fast, and the joint project is completed or abandoned in finite time; whereas in another group in which both players free rides, the joint project is highly inertial, with little learning, and will not be abandoned in finite time. That failing projects of strategic alliances are highly inertial are well documented in the management literature (for instance Doz, 1996).

The above discussion highlights two ingredients for the mutual encouragement effect to arise. First, it is able to counterbalance the deterioration of the background belief. This is guaranteed by Assumption 1, which ensures that a perfect reputation brings in sufficiently good news to encourage U to experiment (given type s_+'s prescribed continuation strategy). Second, it is needed to counterbalance the deterioration of the background belief; the following assumption guarantees this:

Assumption 2. *Signal s_- is sufficiently likely to occur: $\rho_g \geq 1 - \frac{s}{(r+\lambda)h+\lambda h-s}$.*

Under this assumption, without good news channeled in from the strategic component, the fraction of type s_- would eventually become too high for U to continue experimenting before full separation occurs; but then type s_- would have no incentive to mimic type s_+ and would strictly prefer to separate from the latter before full separation occurs, which can't happen in equilibrium. To keep U experimenting at least till the full separation phase, the

[27] To be specific, it does not occur in any MPE that is a limit MPE of a discretization of the continuous-time experimentation game. Hörner, Klein and Rady (2014) (in Lemma 1) show that in any perfect Bayesian equilibrium (hence MPE) of such discrete time game, players do not experiment when their posterior is below the single-player cutoff . Using this result, we can show that in any limit MPE, total effort cannot strictly decrease in players' posterior.

[28] If the fraction of type s_- is high, then we have $p_{gr} > p_1^{*-}$.

strategic component of information has to kick in.[29] We will give a meaning of the bound of ρ_g in the last subsection Section 5.4.1.

With these two assumptions, the mutual encouragement effect can arise, so does the constructed MPE:

Proposition 2. *Under Assumption 1 and 2, an MPE with gradual revelation exists.*

Before explaining the intuitions behind Proposition 1, we flesh out our equilibrium construction.

(1) The Belief System (about I's reputation μ). Low effort completely depletes reputation: if I takes an effort strictly lower than type s_+'s specified effort, he will be taken as type s_-:

$$\mu(s_+|p, \mu, e^I < e^{I+}(p, \mu)) \quad = \quad 0; \tag{11}$$

The belief updating rule for $e^I = e^{I+}(p, \mu)$ is pinned down by Bayes' rule.

(2) Type s_+'s strategy. Type s_+ plays the symmetric MPE strategy under symmetric information as long as his reputation is strictly positive, and a best response — the single-player solution — otherwise. With Assumption 1, this implies he always experiments with full resource before the full separation phase, hence consistent with our equilibrium prescription.

5.1 The rising reputation — the first layer of the mutual encouragement effect

During the gradual revelation phase, I's rising reputation counterbalances the declining background belief, maintaining U's indifference about experimentation, thereby incentivizing her to take interior effort throughout the whole phase. To see this, consider the two main elements that drive U's experimentation incentives.[30]

1. U's expectation about the arriving rate, $\lambda^U(p, \mu)$, defined in page 11. The higher her expectation, the higher her flow marginal benefit of experimentation (MB), $r(\lambda^U h - s)$, and hence the more she is willing to experiment.

2. U's expectation about her ex post future values. Higher expected future value motivates U to speed up experimentation so as to enjoy the future value earlier. That is, the higher expected future value, the higher U's continuation MB.

[29] This is so if type s_+'s effort is 1; due to the strategic substitutability of players' current effort decisions, reducing type s_+'s effort (over time) can also keep U's experimentation incentives.

[30] I's current effort also affects U's experimentation incentive, due to the strategic substitutability of players' current effort decisions, as in the symmetric information game. This element is absent here because I's current effort is 1 with probability 1.

Suppose instead I's reputation does not increase. Then as time passes, U expects her flow MB to be falling, as her expected arriving rate, $\lambda^U(p,\hat{\mu})$, is falling with both the background belief and I's reputation doing so. Moreover, as the background belief deteriorates, U understands both her ex post future values are decreasing, and with I's reputation also dropping, so is her expected future value. Therefore, both elements decrease, undermining U's experimentation incentive. Consequently, if at some point in time she is indifferent about experimentation, she would strictly prefer not to experiment later.

As U is indifferent about experimentation during the gradual revelation phase, I's reputation must rise over time. Indeed, during this phase, I's rising reputation keeps U from becoming pessimistic too quickly about the risky project, and boosts her confidence about having a brighter future (i.e., a higher ex post value). The incentive-enhancing effect of I's rising reputation (driven by the strategic force) exactly balances out the incentive-dampening effect of the deteriorating background belief (driven by the passive force), maintaining U's indifference about experimentation and thus her willingness to take the effort in Figure 5 — in particular, to increase her effort when the background belief is between p_1^{*-} and p_2^{*-}.

Therefore, the first layer of the mutual encouragement effect motivates U to exert appropriate effort over time, so that the second layer of the mutual encouragement emerges.

5.2 U's increasing effort — the second layer of the mutual encouragement effect

When the pessimistic type's belief is between his single-player cutoff p_1^{*-}, and his cooperative cutoff p_2^{*-}, U's increasing effort compensates his growing pessimism, keeping him indifferent between mimicking type s_+ (by continuing experimenting) and revealing himself (by stopping experimenting). As a result, he is both willing to experiment beyond his individual cutoff, and to stop so that the action "continuing experimenting" indeed continually carries encouraging news, propelling the first layer of the mutual encouragement effect.

Specifically, type s_- faces two options. Since the background belief is below his single-player cutoff p_1^{*-}, by revealing himself, he induces both players to stop experimentation; he thereby receives the safe return. By continuing mimicking type s_+ for a dt duration of time, he receives an flow MB, $r\left(\lambda^{I-}(p)\,h - s\right)dt$, which is falling down with the absence of a breakthrough, and a continuation benefit, the upward jump of his continuation value in case a breakthrough arrives, $\lambda h - s$, times its probability $\left(1+e^U\right)\lambda^{I-}(p)$;[31] Therefore, U's effort e^U, through uplifting the chance that type s_-'s continuation value jumps upward, serves as a reward to his hard work. For him to be indifferent about the two options, his total benefit of continuing must match that of stopping; that is, U's effort must satisfy

$$e^U(p,\hat{\boldsymbol{\mu}}(p)) = \frac{r(s-\lambda^{I-}(p)\,h)}{\lambda^{I-}(p)\,(\lambda h - s)} - 1, \text{ for } p \in [p_2^{*-}, \min\{p_1^{*-}, p_{gr}\}]. \tag{12}$$

With the background belief deteriorating, U's effort increases over time. Intuitively, as type

[31] In case no breakthrough arrives, type s_-'s continuation value stays at the safe return s and hence he receives no continuation benefit after this event.

s_-'s MBs from both players' efforts are decreasing and his own effort remains at 1, U's effort — the reward — must increase so that his total net benefit of continuing mimicking to stay at 0.

We thus call this region of the gradual revelation phase the "rewarding region". Observe from Figure 5 that $e^U = 0$ at $p = p_1^{*-}$. This is because, p_1^{*-} being type s_-'s individual cutoff, U does not need to provide any extra reward for him to experiment. Note also that $e^U = 1$ at $p = p_2^{*-}$. This is because, p_2^{*-} being his cooperative cutoff, U needs to respond one for one to type s_-'s (reputation-building) effort, so that type s_- indirectly internalizes the social benefit of his effort at this point.[32] The gradual revelation phase stops here because U reaches the budget limit she can reward I's hard working. U's effort in the other subregion of the gradual revelation phase, (p_1^{*-}, p_{gr}), is left to the final subsection.

Therefore, the second layer of the mutual encouragement effect incentivizes I to build up his reputation, thereby bringing in enough encouraging information to propel the first layer.

5.3 U's growing optimism about the risky project

How much encouraging information should the informed player leaks to U, in order to propel the first layer of the mutual encouragement effect? Lemma 1 gives an answer; U's growing optimism in Proposition 1, follows immediately from the second case.

Lemma 1. *There exists a threshold of the odds ratio $\tilde{a} \in (a^S, \infty)$ such that, during the full pooling phase and the gradual revelation phase of the MPE constructed,*

1. *if $a \in [\tilde{a}, \infty)$, U's expectation of the arrival rate, λ^U, strictly decreases over time;*
2. *if $a \in [a^S, \tilde{a})$, λ^U is U-shaped: it first decreases over time, and then after reaching some point in the gradual revelation region, it begins to increase.*

We are content to give an intuition for why near the end of the gradual revelation phase, U becomes increasingly pessimistic over time if I's initial signal is sufficiently informative ($a \in [\tilde{a}, \infty)$), and increasingly optimistic if it is moderately informative ($a \in [a^S, \tilde{a})$).

Recall that during the gradual revelation phase, U is indifferent between experimenting and not experimenting, which requires her total marginal benefit of experimentation (MB) to be 0. For this to happen, it must be that her flow MB changes in exactly the opposite direction that her continuation MB changes.

U's flow MB is linear in the informed player's posterior beliefs (both when I is of type s_+ and when he is of type s_-). It can be shown that right before full separation, U's continuation MB can be approximated (up to first order) by her expected continuation value if I's type were public and both players played the symmetric MPE.[33] Due to the convexity of player's

[32] The symmetry between players (except asymmetrically informed) plays a crucial role here: in the cooperative solution, the option value of U's experimentation to I is the same with the option value of I's experimentation to U; at p_2^{*-}, type s_- cares about the option value of U's experimentation to him, as if he cares about the option value of his experimentation to U, and hence behave as if he plays cooperatively.

[33] That is, U's continuation MB can be approximated by $\mu w^S(\boldsymbol{q}+(p)) + (1-\mu) w^S(\boldsymbol{q}-(p))$. We will show this in Appendix C.4.3.

continuation value in the symmetric MPE (under symmetric information), U's continuation MB can be taken as convex in I's posterior beliefs (both when I is of type s_+ and when he is of type s_-).

Therefore, a drop in q^- widens the gap between the (ex post) posteriors q^- and q^+, whereby it reduces U's flow MB relatively more than it reduces U's continuation MB. On the contrary, a drop in q^+ shrinks the gap between q^- and q^+, whereby it reduces U's flow MB relatively less than it reduces U's continuation MB.[34]

When the odds ratio a is sufficiently large, q^+ is close to 1 and hence barely decreases over time (by Bayes rule) before full separation occurs and before a breakthrough arrives. The impact of the dropping q^+ on U's MBs is thus dominated by that of the dropping q^-. In this case, although U becomes increasingly pessimistic over time about the risky project, she continues experimenting because she is increasingly convinced that the informed player is optimistic, which raises her continuation value and motivates her to experiment.

When the odds ratio a is intermediate (greater and close to a^S), U is willing to experiment only if I is sufficiently likely to be type s_+, that is, only if I's reputation μ is close to 1.[35] As a result, the dropping q^- ceases to matter because its impact is weighted by $1-\mu$, and the the dropping q^+ dominates. In this case, working for the future value becomes increasingly unappealing; U nonetheless keeps experimenting, because her growing optimism enlivens her. More intuitively, in this case, although U believes I is increasingly likely to be the optimistic type, she understands that the optimistic type himself is getting pessimistic, which disproportionately reduces her continuation MB relative to her flow MB. Fortunately, I's reputation rises fast, so fast that it overturns the deteriorating background belief, rebuilding U's confidence about the risky project and hence her experimentation incentives.

Therefore, we have completed the (sketch of) proof of Proposition 1.

5.4 Detailed equilibrium construction

We now complete our equilibrium construction. Note that type s_+'s strategy and the belief system specified right above Section §5.1 significantly reduce the states over which we need to define strategies, because strategies over the states that are unreachable by any history, in or off equilibrium, does not matter and hence are preferably ignored. The reachable states are

$$\begin{cases} (p,0) \text{ and } (p,\boldsymbol{\mu}^o(p)), & \text{in full pooling phase;} \\ (p,0),(p,\boldsymbol{\mu}^o(p)), \text{ and } (p,\hat{\boldsymbol{\mu}}(p)), & \text{in gradual revelation phase;} \\ (p,0),(p,\boldsymbol{\mu}^o(p)), \text{ and } (p,1), & \text{in full separation phase.} \end{cases}$$

$(p,\boldsymbol{\mu}^o(p))$ occurs either at date 0, or during the full pooling phase; $(p,0)$ occurs if I takes an effort strictly lower than type s_+'s equilibrium strategy; $(p,1)$ if I takes an effort strictly higher than type s_+'s, which is possible only during the full separation phase; and $(p,\hat{\boldsymbol{\mu}}(p))$ on the gradual revelation path. At the beginning of this section, we have defined all strategies

[34] Appendix C.4.3 gives a detailed illustration.

[35] See the discussion of Lemma 3 in the last subsection for further explanation.

over the above states, except during the gradual revelation phase, how exactly type s_- mixes between mimicking type s_+ (by taking effort 1) and stopping mimicking (by playing the symmetric MPE strategy under symmetric information s_-), and how much effort U exerts outside the rewarding region of this phase.

As we have mentioned (in Section 4.4), we will use the changing reputation to describe the above mixed strategy of type s_-. We first describe for a given gradual revelation path $\hat{\boldsymbol{\mu}}$, how type s_- can implement the mixed strategy so that as long as he keeps mimicking his reputation moves along $\hat{\boldsymbol{\mu}}$ in a simple and intuitive Markovian fashion. After this, we derive the necessary and sufficient condition on $\hat{\boldsymbol{\mu}}$ and U's effort during the gradual revelation phase, so that the MPE with gradual revelation is indeed an equilibrium.

First, at date-0 state $(p, \boldsymbol{\mu}^o(p))$ of the gradual revelation phase, type s_- reveals with probability $\boldsymbol{Y}(p)$, such that the action of non-revealing pushes the state up on the curve $\hat{\boldsymbol{\mu}}$, with $\hat{\boldsymbol{\mu}}$ to be determined soon and $\boldsymbol{Y}(p)$ by Bayes' rule :

$$\hat{\boldsymbol{\mu}}(p) = \frac{\boldsymbol{\mu}^o(p)}{\boldsymbol{\mu}^o(p) + [1 - \boldsymbol{\mu}^o(p)]\,(1 - \boldsymbol{Y}(p))}. \tag{13}$$

We've illustrate this in Figure 3: if the state starts on the green dashed line in the gradual revelation region, then type s_- mixes between revealing and not revealing, such that the state immediately jumps on the red line $\hat{\boldsymbol{\mu}}$ if he takes effort 1.[36]

Second, if the state is at $(p, \hat{\boldsymbol{\mu}}(p))$, meaning that conditional on a breakthrough has not arrived, type s_-'s strategy has prescribed him to reveal with cumulative probability $\boldsymbol{Y}(p)$, then type s_- mixes between revealing and not revealing at such an intensity, such that the cumulative probability of separation follows $\boldsymbol{Y}(p')$, for $p' \in (p_2^{*-}, p)$. How does type s_- implement this strategy? Consider a cutoff stopping strategy that specifies a cutoff belief below which type s_- stops mimicking type s_+. His mixed strategy is then a mixture over these cutoff stopping strategies. To implement it, type s_- at date 0 runs a randomization device, according to whose realized value, chooses a cutoff belief to stop mimicking type s_+.[37]

5.4.1 Necessary conditions for equilibrium construction

(1) U's effort function e^U. At any state $(p, \hat{\boldsymbol{\mu}}(p))$ on the gradual revelation path, type s_- faces two options: mimicking type s_+, and not mimicking.

If he mimics, he will receive continuation value $W^{I-}(p, \hat{\boldsymbol{\mu}}(p))$, which satisfies the Hamilton–Jacobi–Bellman (HJB) equation (the argument $(p, \hat{\boldsymbol{\mu}}(p))$ is omitted whenever no confu-

[36] U's strategy is not specified here because what she does at the instant $t = 0$ does not affect her payoff.

[37] Formally, consider a randomization device which uniformly draws a random variable over $[0, 1]$; if $r \in [0, 1]$ is realized, then type s_- will choose to stop mimicking type s_- at cutoff belief $\boldsymbol{Y}^{-1}(r)$, where the superscript -1 refers to inverse of a function. Under this strategy, the cumulative probability of stopping at belief $\tilde{p}$, the probability that $p \geq \tilde{p}$, equals the probability that $\boldsymbol{Y}^{-1}(r) \geq \tilde{p}$ (by the definition of $\boldsymbol{Y}^{-1}$), which equals the probability that $r \leq \boldsymbol{Y}(\tilde{p})$ (as $\boldsymbol{Y}$ is strictly and continuously decreasing), namely, $\boldsymbol{Y}(\tilde{p})$ (as the random variable r is uniformly distributed over $[0, 1]$).

sion arises):

$$
\begin{aligned}
r\left(W^{I-}-s\right) &= e^{I+}\left[r\left(\lambda^{I-}(p)\,h-s\right)-\lambda p\,(1-p)\,\frac{dW^{I-}}{dp}+\lambda^{I-}(p)\left(\lambda h-W^{I-}\right)\right] \\
&\quad +e^{U}\left[-\lambda p\,(1-p)\,\frac{dW^{I-}}{dp}+\lambda^{I-}(p)\left(\lambda h-W^{I-}\right)\right]. \qquad (14)
\end{aligned}
$$

Reputation being interpreted as an asset, this equation simply says type s_-'s "required return", or his flow continuation value, of holding this asset for dt duration of time (the left-hand side of equation (14) times dt), equals the sum of his flow MB times dt,

$$
r\left[e^{I+}\left(\lambda^{I-}(p_t)\,h-s\right)\right]dt,
$$

and the "capital gain" $E[dW^{I-}(p_t,\hat{\boldsymbol{\mu}}(p_t))$. The latter consists of two parts: an upward jump of his continuation value in case a breakthrough arrives, $(\lambda h-W^{I-})$, weighted by its probability $\left(e^{I+}+e^{U}\right)\lambda^{I-}(p_t)\,dt$; and a change of his continuation value in case a breakthrough does not arrive,

$$
\frac{dW^{I-}}{dp}dp_t=-\frac{dW^{I-}}{dp}\left(e^{I+}+e^{U}\right)\lambda p_t\,(1-p_t)\,dt,
$$

weighted by the complementary probability. The continuation MB of experimentation is the capital gain per unit of effort, the terms inside the square bracket of the second line of equation (14).

If he does not mimic, then according to the belief updating rule (11), he will reveal his type, and hence receives $W^{I-}(p,0)$, which is equal to $w^{S}\left(\boldsymbol{q}^{-}(p)\right)$ by the equilibrium construction. $w^{S}\left(\boldsymbol{q}^{-}(p)\right)$ satisfies a similar HJB equation with equation (14), except that e^{I+} and e^{U} are replaced by $e^{S}\left(\boldsymbol{q}^{-}(p)\right)$, the symmetric MPE effort.

Since type s_- is indifferent between these two options during the gradual revelation phase, we have $W^{I-}(p,\hat{\boldsymbol{\mu}}(p))=w^{S}\left(\boldsymbol{q}^{-}(p)\right)$. This implies, first, his continuation MBs from each players' effort under the two options are the same, because the continuation MBs only depend on continuation value functions (referring again to the terms inside the square bracket of the second line of equation (14)). Second, his total MBs under the two options are also the same, as the flow MBs depend only on his posterior. This means, whenever he is indifferent about experimentation right after revealing himself, as what happens during the gradual revelation phase but outside the rewarding region — $(p_1^{*-},p_{gr}]$, he would also be indifferent about experimentation if whatever effort he takes does not impair his reputation; in such circumstances, mimicking type s_+ costs nothing and hence type s_- shouldn't be compensated by any reward for doing so. Therefore, a third implication follows: during the gradual revelation phase but outside the rewarding region, type s_-'s total benefit from U's effort under the two options are the same . Since his MB from U's effort are also the same, so should U's effort levels under both options. This means, U does not reward type s_-'s hardworking over $(p_1^{*-},p_{gr}]$ of the gradual revelation phase, which is thus called the non-responding region.

The non-responding region does not alway exist, that is, p_{gr} might be lower than p_1^{*-}. For a given odds ratio, if the fraction of type s_- is low, or ρ_b is low, then U is willing to experiment even at low background beliefs, implying a short gradual revelation phase, or a low p_{gr}. It can be shown that for each odds ratio a satisfying Assumption 1, there is a threshold of ρ_b above which, the non-responding region exists, and below which, it does not; below this threshold, as ρ_b decreases, the rewarding region begins to shrink and as ρ_b reaches a lower threshold, it ceases to exist. Assumption 2 is sufficient condition under which the gradual revelation phase exists.

Lemma 2 summarizes our result, together with U's effort during the rewarding region.

Lemma 2. *Assume the MPE with gradual revelation is an equilibrium. Then,*

- *Over the Rewarding Region* $(p_2^{*-}, \min\{p_1^{*-}, p_{gr}\}]$ *of the gradual revelation phase, U's effort satisfies equation* (12) *along the gradual revelation path, and hence is strictly increasing over time.*
- *Over the non-responding Region* $(\min\{p_1^{*-}, p_{gr}\}, p_{gr}]$ *(if nonempty) of the gradual revelation phase, U's effort equals the symmetric MPE effort under public information* s_-*, and hence is strictly decreasing over time.*

Referring to Figure 5 again, U's effort is decreasing over time over the non-responding region (p_1^{*-}, p_{gr}),[38] and increasing over the rewarding region (p_2^{*-}, p_1^{*-}). Although U's effort is U-shaped (if the non-responding region exists), her reward, measured by the discrepancy between her efforts before and after type s_- reveals himself, $(e^U(p, \hat{\boldsymbol{\mu}}) - e^U(p, 0))$, is monotonically increasing over time: it stays at 0 when mimicking is costless, and begins to rise when it becomes harder.

(2) Type s_-'s revealing strategy $\hat{\boldsymbol{\mu}}$. From Lemma 2, U's effort is interior and hence she is indifferent about experimentation during the whole gradual revelation phase; we now analyze what condition $\hat{\boldsymbol{\mu}}$ should satisfy to keep U's indifference.

In a dt time interval, three events can happen: a breakthrough does not arrive, a breakthrough arrives, and I stops mimicking type s_+, each with a probability depending on dp, molding U's experimentation incentives. The first two events contribute to U's continuation MB as they do to type s_-'s. The third event happens with probability $(1-\mu_t)\,\frac{dY_t}{1-Y_t}$ (to U), and when it happens, reduces U's continuation value by $|W^U(p,0) - W^U(p,\hat{\boldsymbol{\mu}})|$. If U takes effort e during a dt interval, then using equation (9) and $d\mu_t = \hat{\boldsymbol{\mu}}_p dp_t$, this probability can be expressed through the state variables:

$$(1-\hat{\boldsymbol{\mu}}(p_t))\frac{dY_t}{1-Y_t} = \left(\phi(p_t,\mu_t) - \frac{\hat{\boldsymbol{\mu}}_p(p_t)}{\hat{\boldsymbol{\mu}}(p_t)}\right)(e_t^I + e)p_t(1-p_t)\lambda dt, \tag{15}$$

[38] Note although U reduces her effort, and hence remains indifferent about experimentation in both this non-responding region of the asymmetric information game and in the symmetric information benchmark, she does so out of different reasons: in the former, she is indifferent because I's rising reputation compensates the absence of breakthrough , whereas in the latter, I's dropping effort compensates the absence of a breakthrough (due to the substitutability of players' current effort decisions).

where $\phi\left(p_t, \mu_t\right) \equiv \frac{(1-\mu_t)(\rho_b-\rho_g)}{\boldsymbol{\mu}^o(p_t)(1-\boldsymbol{\mu}^o(p_t))}$.

U's continuation MB of experimentation, denoted as $A(p, \hat{\boldsymbol{\mu}})$, sums up the weighted effects of the three events:

$$\begin{aligned} A(p, \hat{\boldsymbol{\mu}}) &\equiv -\lambda p\left(1-p\right) \frac{dW^U\left(p, \hat{\boldsymbol{\mu}}\right)}{dp} + \lambda^U\left(p, \hat{\boldsymbol{\mu}}\right)\left(\lambda h - W^U\left(p, \hat{\boldsymbol{\mu}}\right)\right) \\ &\quad + \left(\phi\left(p, \hat{\boldsymbol{\mu}}\right) - \frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}\right) p(1-p)\lambda\left(W^U\left(p, 0\right) - W^U\left(p, \hat{\boldsymbol{\mu}}\right)\right). \end{aligned} \tag{16}$$

Her continuation value function $W^U(p, \hat{\boldsymbol{\mu}}(p))$ thus satisfies the HJB equation

$$r\left(W^U\left(p, \hat{\boldsymbol{\mu}}\right) - s\right) = \max_{e \in [0,1]} e\left[r\left(\lambda^U\left(p, \hat{\boldsymbol{\mu}}\right) h - s\right) + A\left(p, \hat{\boldsymbol{\mu}}\right)\right] + A\left(p, \hat{\boldsymbol{\mu}}\right), \tag{17}$$

where the left-hand side is U's flow net continuation value. Her indifference about experimentation during the gradual revelation phase implies her total MB equal to 0:

$$r\left(\lambda^U\left(p, \hat{\boldsymbol{\mu}}\right) h - s\right) + A\left(p, \hat{\boldsymbol{\mu}}\right) = 0. \tag{18}$$

Equations (17) and (18) further imply that U's net continuation value equals her flow MB experimentation divided by r:

$$W^U\left(p, \hat{\boldsymbol{\mu}}\right) - s \quad = \quad s - \lambda^U\left(p, \hat{\boldsymbol{\mu}}\right) h. \tag{19}$$

U's indifference condition (18) and her continuation value function (19) imply that the gradual revelation path $\hat{\boldsymbol{\mu}}$ satisfies the following ODE:

$$\hat{\boldsymbol{\mu}}_p = g(p, \hat{\boldsymbol{\mu}}), \qquad p \in (p_2^{*-}, p_{gr}), \tag{20}$$

where the formula of g is given in Appendix C.2.3 due to its complexity and lack of direct intuition.

The following lemma characterizes the gradual revelation path $\hat{\boldsymbol{\mu}}$.

Lemma 3. *Let the MPE with gradual revelation be an equilibrium. Then the gradual revelation path $\hat{\boldsymbol{\mu}}$ is the unique solution to the first order ODE problem defined by equation* (20)*, with the initial value condition*

$$\hat{\boldsymbol{\mu}} = \frac{s - \lambda^{I-}\left(p\right) h}{W^{I+}\left(p, 1\right) - s + \lambda^{I+}\left(p\right) h - \lambda^{I-}\left(p\right) h}, \qquad p = p_2^{*-}, \tag{21}$$

and the boundary p_{gr} being the smallest p satisfying

$$\hat{\boldsymbol{\mu}}\left(p_{gr}\right) = \boldsymbol{\mu}^o(p_{gr}), \tag{22}$$

The initial condition comes from the value matching condition of W^U at p_2^{*-}:

$$s - \lambda^U(p, \hat{\boldsymbol{\mu}})\, h = \hat{\boldsymbol{\mu}} \left(W^{I+}(p, 1) - s \right), \qquad p = p_2^{*-}, \tag{23}$$

where the left-hand side is U's equilibrium value (19) at p_2^{*-} whereas the right-hand side U's expected value right before fully separating, the expectation of her ex post values after revelation, positive only if I is type s_+. Finally, since after the full pooling phase I's reputation μ_t must *gradually rise* above $\boldsymbol{\mu}^o(p_t)$ as long as type s_- does not reveal, we must have, first, the gradual revelation path is above the full pooling path at p_2^{*-}, $\hat{\boldsymbol{\mu}}(p_2^{*-}) > \boldsymbol{\mu}^o(p_2^{*-})$. Assumption 2 guarantees this: the bound in this assumption equals the lowest possible level of reputation right before the gradual revelation phase ends, obtained when the optimistic type knows the risky project is good; since the fraction of the optimistic type under full pooling $\boldsymbol{\mu}^o$ is always lower than ρ_g, we have $\hat{\boldsymbol{\mu}}(p_2^{*-}) > \boldsymbol{\mu}^o(p_2^{*-})$ under Assumption 2.

Second, the right boundary of the gradual revelation phase, p_{gr}, is the smallest background belief at which the gradual revelation path $\hat{\boldsymbol{\mu}}$ and the full pooling path $\boldsymbol{\mu}^o$ intersects.

Remark. The initial condition says, the value of $\hat{\boldsymbol{\mu}}$ right before the full separation phase decreases in the the odds ratio a. Intuitively, the greater the odds ratio, the bigger gap between the two types' beliefs, and hence the higher her ex post continuation value in case I is type s_+, implying a higher incentive for U to experiment so as to collect this value earlier.

5.4.2 Sufficient conditions for equilibrium construction and existence

The necessary conditions provided in the previous subsection is also sufficient for the MPE with gradual revelation to be an equilibrium:

Lemma 4. *The MPE with gradual revelation is an equilibrium, if during gradual revelation, the uninformed player's effort is as in Lemma 2, and that type s_-'s revealing intensity is such that, the associated revealing path $\hat{\boldsymbol{\mu}}$ is a (unique) solution to the ODE problem defined by* (20), (21), *and* (22).

The existence of an MPE with gradual revelation, presented in Proposition 2, follows from Lemma 4.

6 Multiplicity of Equilibria

6.1 Equilibrium refinement

Not surprisingly, the asymmetric information game has multiple MPEs, both due to the strategic substitutability of current effort decisions in the experimentation game itself, and the arbitrariness of assigning off-equilibrium beliefs in a signaling game. To select sensible equilibria, we focus on MPEs that survive a criterion in the spirit of the D1 criterion, and that players play the symmetric MPE under symmetric information, after the uninformed player learns the true type of the informed player. We call the former restriction D1, and the latter restriction SMPE for simplicity. Moreover, we require equilibrium strategies to be left continuous with respect to the background belief.

Even so, one source of multiplicity is unavoidable: when the optimistic type's posterior q^+ is in (q_1^*, q^S), the interior experimentation region of the symmetric MPE if his signal were public, the uninformed player is willing to experiment right before full separation if the informed player experiments too little.[39] Then the optimistic type would have no incentive to deviate to get a perfectly bad or perfectly good reputation, since in either case, he loses the chance to free ride. Therefore, D1 does not have a bite here. Nonetheless, when the belief gap between the two types is large enough, we must have full separation over background beliefs $[p_2^{*-} - \epsilon, p_2^{*-}]$ for a small ϵ.[40] The reason is that the above-mentioned pooling requires the informed player to exert low effort for a long time,[41] which would lead to a lower continuation value to type s_+ than the symmetric MPE under public information s_+ does; in such circumstances, type s_+ strictly prefers to deviate to a high effort, since by D1, he would get a perfectly good reputation for doing so. Using this result and D1, we can pin down the equilibrium path when the prior q_0 lies above p_2^{*-}:

Claim 1. *There exists a threshold $\bar{a}$, such that if the odds ratio a is greater than $\bar{a}$ and the prior q_0 is above p_2^{*-}, then the equilibrium path of any MPE satisfying D1 and SMPE coincides with that of the MPE with gradual revelation constructed in Section 5.*

The proof is in Appendix E.1. The intuition behind this result is that, if there is no reputation concerns, then type s_+ strictly prefers to experiment even if U's effort is 1 (recall that current effort decisions are strategic substitutes), whereas type s_- either strictly prefers not to experiment (over the rewarding region) or is indifferent (over the non-responding region) when U's effort coincides with the interior effort along $(p, \hat{\boldsymbol{\mu}})$. Therefore, the reason that type s_+ might choose effort lower than 1 in some MPE must be that effort 1 leads to a reputation under which, in the future, either U free-rides or he himself is required to free-ride. To rule out such possibilities, for a given MPE, we first identify the smallest p (above p_2^{*-}) around which its type s_+'s equilibrium effort differ from that of the MPE constructed in Section 5; we then show that the set of reputation making type s_+ strictly benefits from deviating to effort 1 is strictly larger than the set of reputation making type s_- weakly benefits from such a deviation. Therefore, by D1, whenever I deviates to effort 1 over $[p, p + dp]$, he should receive a perfect reputation. But then type s_+ indeed has a profitable deviation; hence the MPE under consideration cannot survive the D1 criterion. This reasoning implies that type s_+'s equilibrium effort must be 1 for all $p > p_2^{*-}$, in any MPE satisfying D1 and SMPE.

Assumption 1 is not enough for the result in Claim 1 to hold because although the optimistic type has strict incentives to experiment at all p's that are greater than p_2^{*-}, he

[39] The uninformed player is willing to experiment alone as long as her continuation value is higher than the safe return. Hence if $\mu > 0$ and the duration of time she experiments alone before full separation is not too long, her continuation value will indeed be higher than s.

[40] Recall that p_2^{*-} is the boundary between the full separation phase and the gradual revelation phase of the MPE with gradual revelation.

[41] Otherwise, type s_- would strictly prefer to separate.

might still exert low effort, in fear that if the uninformed player learns that he is the optimistic type, the uninformed player would exert less effort in the future.[42]

6.2 MPEs without Assumption 1

If Assumption 1 does not hold, then there are multiple MPEs satisfying D1, SMPE, and close to the MPE with gradual revelation constructed in Section 5. Typically, such an MPE has an additional pooling phase, which occurs between its gradual revelation phase and its full separation phase.[43] That is, an MPE has four phases: full pooling when p is high; gradual revelation when p is moderately high; pooling when p is moderately low;[44] and full separation when p is low. When Assumption 1 does not hold, a sufficient condition for U's rising effort pattern in Proposition 1 to occur in some MPE is that the odds ratio a is greater than $\frac{q^S}{1-q^S}/\frac{q_1^*}{1-q_1^*}$. When the odds ratio is too small, then such rising effort patterns (over time) do not occur in MPEs close to the MPE with gradual revelation constructed in Section 5.[45]

An MPE with only pooling and full separation phases always exist. We focus on MPEs with gradual revelation because it delivers new insights, and qualitatively different behavior and belief dynamics. Moreover, the MPE with a gradual revelation phase during which the optimistic type's effort is 1 corresponds to the extreme such that, the informed player's private information is revealed as quickly as possible (in terms of the background belief), and the uninformed player's experimentation incentive is maintained by such information. Pooling equilibria represent another extreme, in which the informed player's private informa-

[42]The pooling discussed above Claim 1, that is, pooling at background beliefs below p_2^{*-}, is unappealing for two reasons. First, the ex-post total welfare of such a pooling MPE over background beliefs $[0, p_2^{*-}]$ is lower than the ex-post total welfare of some MPE in which full separation occurs over $[0, p_2^{*-}]$. This is because in case I holds signal s_-, experimenting at $p < p_2^{*-}$ generates a lower total welfare than not experimenting, as p_2^{*-} is the cooperative cutoff; in case I holds signal s_+, type s_+ can be prescribed to play his effort on the pooling path of the pooling MPE, and U can be prescribed to weakly higher effort than her effort on the pooling path of the pooling MPE. Second, there is nothing new in the pooling mechanism: U's experimentation incentives are sustained by I's low effort as her posterior deteriorates over time, which is exactly the same as in an asymmetric MPE of the symmetric information game.

If we impose full separation for background beliefs below p_2^{*-}, then Assumption 1 would be sufficient for the result in Claim 1 to hold. A sufficient condition for full separation to occur at background beliefs below p_2^{*-} is $e^U(p, \mu) \leq e^U(p, 1)$, which restricts the punishment role of a perfectly good reputation.

[43]I still call the region where both players stop experimenting ($[0, p_1^{*+}]$) full separation, because type s_+ could exert a positive effort at $t = 0$ (right after learning his type) and reveal his type. Such an action is costless in continuous time.

[44]This new pooling phase occurs over a subset of $[\max\{p_2^{*-}, p_1^{*+}\}, p^{S+})$. When Assumption 1 does not hold, a gradual revelation phase cannot lasts till $\max\{p_2^{*-}, p_1^{*+}\}$. This is because, for gradual revelation to occur, the uninformed player's effort must be interior and strictly lower than the optimistic type's effort; the former implies that both U's effort and the optimistic type's are lower than the symmetric MPE effort under symmetric information s_+, and hence the optimistic type has strict incentive to experiment. The MPE (with the above-mentioned feature) cannot satisfies D1, because the optimistic type has a higher incentive than the pessimistic type to deviate and show that he is indeed a optimistic type, so as to induce both players to work harder.

[45]That is, the rewarding region disappears.

tion is withhold as late as possible, and the uninformed player's experimentation incentive is maintained by lowering the informed player's effort (over time). Between these two extreme, we can construct a continuum of hybrid MPEs.[46]

7 Welfare Analysis

We now examine the welfare impact of asymmetric information. We compare players' ex ante total payoff at the initial background belief p_0 in the MPE with gradual revelation constructed in Section §5, shortened as the "ex ante total welfare under asymmetric information":[47]

$$W^U(p_0, \boldsymbol{\mu}^o(p_0)) + \boldsymbol{\mu}^o(p_0)W^{I+}(p_0, \boldsymbol{\mu}^o(p_0)) + (1 - \boldsymbol{\mu}^o(p_0))W^{I-}(p_0, \boldsymbol{\mu}^o(p_0)), \tag{24}$$

and that in the symmetric MPE of the symmetric information game, shortened as the "ex ante total welfare under symmetric information":[48]

$$2\boldsymbol{\mu}^o(p_0)w^S(\boldsymbol{q}^+(p_0)) + 2(1 - \boldsymbol{\mu}^o(p_0))w^S(\boldsymbol{q}^-(p_0)). \tag{25}$$

Denote $\Delta W(p_0)$ the discrepancy between these two welfares, that is, expression (24) minus expression (25). We will say:

Definition 1. Asymmetric information

- improves ex ante total welfare at p_0 if and only if $\Delta W(p_0) > 0$;
- deteriorates ex ante total welfare at p_0 if and only if $\Delta W(p_0) < 0$.

Asymmetric information generates a benefit because, due to the mutual encouragement effect, in case I received signal s_-, players experiment more than they would do in the symmetric information benchmark. Asymmetric information may however, incur a cost; in case I received signal s_+, then during the gradual revelation phase, U experiments less than in the symmetric information benchmark, resulting in an ex post total welfare loss. This implies, if ρ_b is so low, and hence the fraction of type s_- is so small, that the gradual revelation phase of the MPE with gradual revelation in Section 5 disappears,[49] then asymmetric information does not incur any cost. As a result, asymmetric information improves ex post individual welfares, and hence it improves ex ante total welfare. In the sequel we focus on the nontrivial case such that the gradual revelation phase exists. Proposition 3 presents the main welfare result. Since asymmetric information does not affect welfare over the full separation region, we only present the welfare results over the gradual revelation region and the full pooling region, $(p_2^{*-}, 1)$.

[46] "Hybrid" is in the sense that the uninformed player's experimentation incentives are maintained by gradual revelation and lowering the informed player's effort at the same time.

[47] Note that the initial background belief p_0 equals the initial common prior q_0.

[48] That is, it is the expected total welfare at the initial background belief, before player I's signal is realized and becomes public.

[49] Assumption 2 in Proposition 2 is only a sufficient condition for the gradual revelation phase to exist. See the discussion right above Lemma 2 for a sufficient and necessary condition on ρ_b.

Proposition 3. *Under Assumption 2,*

- *if $a \in [1 + 2\frac{\lambda}{r}, \infty)$, then asymmetric information improves ex ante total welfare over the gradual revelation region and the full pooling region $(p_2^{*-}, 1)$.*
- *if $a \in [a^S, 1 + 2\frac{\lambda}{r})$, then asymmetric information deteriorates ex ante total welfare over some p_0's.*

Interested readers can refer to Proposition 4 presented at the end of this section for a detailed welfare characterization when $a \in [a^S, 1 + 2\frac{\lambda}{r})$.

The main message that Proposition 3 delivers is that, under Assumption 1 and 2, asymmetric information improves ex ante total welfare (except over the full separation region) if and only if I's initial signal is sufficiently informative, or equivalently, if and only if the degree of asymmetric information is large enough. We now explain its intuition. We will give a meaning to the threshold $1 + 2\frac{\lambda}{r}$ afterward.

Compared with symmetric information, in the asymmetric information game, type s_+ exerts the same level of effort, type s_- more effort; U exerts less effort than in the symmetric MPE when s_+ is public, and more when s_- is public. Therefore, at the interim stage (right after I learns his type),

(i) type s_+ suffers from asymmetric information because he does not enjoy as much informational externality as he does in the symmetric benchmark due to U's lower effort, except when $\rho_b = 1$. If $\rho_b = 1$ (or, a is infinity), then type s_+ knows the risky project is good and hence there is nothing for him to learn.

(ii) Type s_- does not suffer or benefit from asymmetric information in the gradual revelation phase, and benefits from it in the full pooling phase due to U's harder work than in the symmetric benchmark.

(iii) U benefits from asymmetric information. Conditional on a breakthrough has not arrived and type s_- has not revealed himself, U has the option of matching her effort level to I's. Doing so, in case I holds signal s_+, both players would experiment as they would do in the symmetric information setup with s_+ public, whereby U achieves the same ex post continuation value as under symmetric information. In case I holds signal s_-, both players would experiment more than they would do in the symmetric information setup with s_- public; as experimentation is desirable for the two players over the gradual revelation region, and they equally share the workload, U achieves a strictly greater continuation value than under symmetric information. Therefore, this effort-matching option (in the MPE constructed) guarantees her a higher interim value than does the symmetric benchmark. If she does not take this option in the MPE constructed, it must be that she is better off in this MPE, and hence better off than under symmetric information.

Type s_+'s loss from asymmetric information is decreasing in the odds ratio, whereas the two players' gain in case I received signal s_- is increasing in it. Intuitively, first, the greater the belief difference, the more optimistic type s_+ is during the gradual revelation region, and hence the less type s_+ needs to learn from U's experimentation outcome, consequently the smaller loss type s_- suffers from asymmetric information.[50] Second, the greater the belief difference, the less type s_- needs to reveal to compensate player U during the gradual revelation region, and hence the higher probability that the two players still continue experimenting in the future, implying a greater gain to them in case I holds signal s_- .

In one extreme when the odds ratio a is infinity (occurring if $\rho_b = 1$), type s_+ does not suffer any loss from asymmetric information whereas type s_- and U strictly benefit from it; asymmetric information thus improves ex ante total welfare unambiguously. In the other extreme when the odds ratio a equals a^S, U is willing to experiment at the end of the gradual revelation phase p_2^{*-} only if she believes her teammate is type s_+ for sure, or, $\hat{\boldsymbol{\mu}} = 1$ at p_2^{*-}; as a result, type s_+'s loss dominates, and asymmetric information deteriorates ex ante total welfare (at least for background belief close to p_2^{*-}). By continuity and the monotonicity of the ex post gains and losses, there is a threshold of the odds ratio, above which asymmetric information improves ex ante total welfare universally and below which it does not.

We are thus done with the main message. What is the meaning of the threshold $1 + 2\frac{\lambda}{r}$? It turns out, it equals $\frac{q^M}{1-q^M} / \frac{q_2^*}{1-q_2^*}$, where q^M is a player's myopic cutoff (in true posterior). That is, it is the odds ratio under which, when type s_-'s posterior is at his cooperative cutoff q_2^* (hence the end of the gradual revelation phase), type s_+'s is at exactly his myopic cutoff q^M. We now give an intuition why this is so, for interested readers.

First, the myopic cutoff q^M is special not only because it is the optimal cutoff for a myopic player, but also because it is the threshold (in true posterior) above which, a player's flow continuation value $r(w^S - s)$ under the symmetric information benchmark is greater than the *social* continuation MB of effort, and below which it is smaller. To see this, consider a player's HJB equation around q^M under the symmetric information benchmark (with state variable being the common posterior q), as both players take effort 1 around q^M, we have

$$\underbrace{r(w^s - s)}_{\text{flow continuation value}} \quad = \quad \underbrace{r(\lambda q h - s)}_{\text{flow MB}} + 2 * \text{continuation MB}.$$

Since there are two symmetric players playing the symmetric MPE, twice the continuation MB to one player is also the social continuation MB of effort.

The above HJB equation says a player's flow continuation value in the symmetric information benchmark is greater than the social continuation MB if and only if his or her posterior is greater than the myopic cutoff posterior. In other words, the social continuation loss due to U's low effort in a dt time interval, in case I received signal s_+, is smaller than

$$r(1 - e^U)(w^s(q^+) - s)dt \tag{26}$$

right before fully separation if and only if $q^+ \geq q^M$, or equivalently, $a \geq a^M$.

[50] Here we use the property that in the symmetric MPE under symmetric information, before experimentation stops, the option value per unit of effort to a player is increasing over time as his or her posterior decreases. Intuitively, the more a player's posterior differs from 1, the more informational value a breakthrough is (that is, a player's continuation value is convex).

Second, under asymmetric information, apart from U's continuation loss caused by her own insufficient effort during the gradual revelation phase, her additional gain from asymmetric information, as we now show, is exactly expression (26), right before fully separation. Since type s_- does not benefit or suffer from asymmetric information, combining with the previous paragraph, we obtain that asymmetric information improves ex ante total welfare right before fully separation if and only if $a \geq a^M$. What's U's additional gain from asymmetric information?

- In case I received s_-, compared with type s_-, who obtains the same value as under the symmetric benchmark, U exerts $(1 - e^U)$ less effort, thereby saving flow effort cost $r(1 - e^U)(s - \lambda^{I-}h)dt$. Since the continuation benefit of effort are the same for both, this is also her additional ex post gain from asymmetric information.

- In case I received s_+, U exerts $(1 - e^U)$ less effort than type s_+, and hence saves flow effort cost $r(1 - e^U)(s - \lambda^{I+}h)dt$.

Therefore, in expectation, in addition to the continuation loss due to her low effort, U enjoys an extra gain of

$$r(1 - e^U)(s - \lambda^U h)dt,$$

which, applying her continuation value formula (19), equals

$$r(1 - e^U)(W^U - s)dt.$$

Applying the value matching condition of W^U right before fully separation, equation (23), we are done.

We now present detailed welfare results for the second case of Proposition 3.

Proposition 4. *Assume $a \in [a^S, 1 + 2\frac{\lambda}{r})$, and Assumption 2 is satisfied. Then*

1. *either asymmetric information deteriorates ex ante total welfare over the gradual revelation region and the full pooling region $(p_2^{*-}, 1)$;*

2. *or asymmetric information strictly deteriorates ex ante total welfare over (p_2^{*-}, p^{S-}), and does not affect it over $[p^{S-}, 1]$;*

3. *or there exists a threshold of the background belief $\tilde{p} \in (p_2^{*-}, p^{S-})$, such that asymmetric information deteriorates ex ante total welfare over $(p_2^{*-}, \tilde{p})$, and improves it over $(\tilde{p}, 1)$.*

A sufficient and necessary condition for the third case to occur is ρ_b being either sufficiently large, or sufficiently small. Intuitively, when ρ_b is sufficiently large, then there is a large fraction of type s_-, implying the two players' expected gain (occurring only in case I holds signal s_-) is large, relative to their expected loss (occurring only in case I holds signal s_+); when ρ_b is sufficiently small, then there is a small fraction of type s_-, implying a short gradual revelation phase, and hence a small expected loss of the two players (occurring only during the gradual revelation phase), relative to the their expected gain.

8 Empirical Implications

We now explore the empirical implications of the joint behavior pattern during the gradual revelation phase of the MPE constructed: in the absence of a breakthrough, in the rewarding region of the gradual revelation phase, the informed player maintains high effort, and the uninformed player increases her effort. The expected total effort can also increase.[51]

First, as we have mentioned, an econometrician unaware of the information asymmetry between players would update his belief — and also think the players update beliefs — according to the background belief. He then would hypothesize that players' total effort decrease over time in the absence of breakthrough. Estimating the (misspecified) symmetric information experimentation model can thus lead him to incorrectly reject the existence of learning through experimentation, or underestimating its effect. Indeed, many empirical papers, Conley and Udry (2010, page 59, Column B, Table 6) for instance, interpret players' non-response to news as the absence of learning through experimentation.[52]

We can distinguish our model from models with myopic agents or from models with symmetric information by investigating a given player's reaction to the introduction of a new player into the game. If a player is myopic, introducing a new player with similar or less experience would not affect his behavior; in contrast, if a player cares about the future, it would, and it would affect his behavior differently under an asymmetric information setup and under a symmetric information setup. First, if a player is experimenting before the introduction of a new player, introducing a similarly experienced player would reduce his effort, due to free riding, whereas introducing a less experienced player would not change much of his behavior (at least in the short run).[53] Second, if he has already stopped experimenting before the introduction and the new player cannot observe whether he has stopped before the introduction but can afterward, then introducing a less experienced player would induce him to restart experimentation.[54]Finally, if two similarly experienced players have already stopped experimentation, then replacing one player with a less experienced player would restart the other player's experimentation, if the new player cannot observe whether the old players have stopped experimenting before the introduction but can observe it afterward.

[51]The expected total effort can increase during the gradual revelation phase if player's discount rate r is not too small, so that the uninformed player's effort rises relatively faster than the pessimistic type reveals himself (over time).

[52]In Conley and Udry (2010), it makes sense to reject the existence of learning through experimentation based on the "non-responsiveness" result in Column B, Table 6; this is because the "technology" being tested is the labor use of an established crop. Without such contexts, the rejection might be incorrect.

[53]The former corresponds to a symmetric information setup, whereas the latter to an asymmetric information setup.

[54]If the good news is not conclusive, then introducing a similar experienced player can also restart a current player's experimentation.

9 Conclusion

This paper has studied the impact of initial information asymmetry on agents' experimentation behavior, using the canonical exponential-bandit model. It has shown that information asymmetry leads to new and interesting joint behavior dynamics. When the public information generated from experimentation becomes too discouraging, the informed player leads by example — exerting high effort despite the unfavorable public information, which motivates the follower (the uninformed player) to work harder over time. The follower's positive response to the leader's high effort encourages the leader also to work hard. Due to this mutual encouragement effect, creating an information asymmetry between two otherwise symmetric agents may improve total welfare.

The welfare analysis has implications for information design in experimentation games. Consider a general setting in which only a social planner can observe the initial signal (of I's), and before the initial signal is realized, she can send to each player as a signal a garbling of the initial information structure. It can be shown that when the informativeness of the initial signal is not too low (that is, it satisfies Assumption 1), the social planner can implement the following outcome: when s_+ is realized, both players play the symmetric MPE (under public information s_+); when s_- is realized, both players play cooperatively. She can do so by: always revealing the true signal (s_+ or s_-) to one player; revealing privately the true signal to the other player with probability $\pi \in (0, 1)$ if s_+ occurs, and revealing nothing otherwise. This strategy gives a strictly higher total payoff than revealing her signal to neither player, to one player, or to both players does.[55] By giving all the relevant information to one player (he), his pessimistic type can be incentivized to stop experimenting at the cooperative cutoff belief; by revealing only the positive signal s_+ to the other player (she) with some probability, her uninformed type can be incentivized to mimic type s_+ before the informed player stops. Therefore, creating an informed leader mitigates free-riding without any risk of over-experimentation. In environments where agents are heterogeneous (in their experimentation benefit or cost, for instance), creating an informed leader can lead to over-experimentation. It would be interesting to study the optimal information design in such environments.

[55] There might exist other strategies (of the social planner) that outperform this strategy in terms of ex-ante total welfare. We do not explore the possibility here because even with this strategy, it is easy to construct an MPE that gives a higher total payoff than the implemented outcome given above.

A Background Belief

This section defines the background belief formally.

Denote $\Omega \equiv \{0,1\} \times \{s_+, s_-\} \times \Omega_N$, where Ω_N is the set of point process paths. Similarly denote $\Omega^t \equiv \{0,1\} \times \{s_+, s_-\} \times \Omega_N^t$, where Ω_N^t is the set of point process paths till time t. Let $\sigma(\{0,1\} \times \{s_+, s_-\})$ be the sigma-algebra generated by $\{0,1\} \times \{s_+, s_-\}$, $(\mathcal{F}_t^N)_t$ the filtration generated by point process N. Define $(\mathcal{F}_t)_t \equiv (\sigma(\{0,1\} \times \{s_+, s_-\}) \otimes \mathcal{F}_t^N)_t$ and $\mathcal{F} \equiv \mathcal{F}_\infty$. For a given prior p_0, an effort path $e \equiv (e_t^I, e_t^U)_{t\geq 0}$ induces a distribution P_{e,p_0} over the filtered space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0})$, satisfying for each $(\theta, s_l, N^t) \in \Omega^t$ where N^t denoting the experimentation outcome history such that no breakthrough has arrived till time t,[56]

$$\begin{aligned} P_{e,p_0}(\theta, s_l, N^t) &= P_{e,p_0}(s_l, N^t|\theta) P_{e,p_0}(\theta) \\ &= P_{e,p_0}(s_l|\theta) P_{e,p_0}(N^t|\theta) P_{e,p_0}(\theta), \end{aligned}$$

where the second inequality is due to the fact that given e and conditional on θ, s_l and N^t are independently distributed. Note that $P_{e,p_0}(\theta) = p_0$, $P_{e,p_0}(s_l|\theta)$ does not depend on e and p_0, and $P_{e,p_0}(N^t|\theta)$ does not depend on p_0.

Given P_{e,p_0}, the distribution of θ conditional on (s_l, N^t), if $\sum_\theta P_{e,p_0}(s_l|\theta) P_{e,p_0}(N^t|\theta) P_{e,p_0}(\theta) > 0$, is

$$P_{e,p_0}(\theta|s_l, N^t) = \frac{P_{e,p_0}(s_l|\theta) P_{e,p_0}(N^t|\theta) P_{e,p_0}(\theta)}{\sum_\theta P_{e,p_0}(s_l|\theta) P_{e,p_0}(N^t|\theta) P_{e,p_0}(\theta)} \tag{27}$$

In our asymmetric information game, $P_{e,p_0}(\theta|s_l, N^t)$ is the s_l-type informed player's posterior, after he observes effort history e^t and experimentation outcome history N^t.

(1) We now show that for a given effort path e, this posterior does not depend on whether he observes s_l before N^t or after.

Dividing both the numerator and denominator of the right-hand side of equation (27) by $\sum_{\tilde{\theta}} P_{e,p_0}(s_l|\tilde{\theta}) P_{e,p_0}(\tilde{\theta})$, we have

$$\begin{aligned} P_{e,p_0}(\theta|s_l, N^t) &= \frac{P_{e,p_0}(N^t|\theta)[P_{e,p_0}(s_l|\theta) P_{e,p_0}(\theta) / \sum_{\tilde{\theta}} P_{e,p_0}(s_l|\tilde{\theta}) P_{e,p_0}(\tilde{\theta})]}{\sum_\theta P_{e,p_0}(N^t|\theta)[P_{e,p_0}(s_l|\theta) P_{e,p_0}(\theta) / \sum_{\tilde{\theta}} P_{e,p_0}(s_l|\tilde{\theta}) P_{e,p_0}(\tilde{\theta})]} \\ &= \frac{P_{e,p_0}(N^t|\theta) P_{e,p_0}(\theta|s_l)}{\sum_\theta P_{e,p_0}(N^t|\theta) P_{e,p_0}(\theta|s_l)} \end{aligned}$$

The second equality is by Bayes rule. This equality can be interpreted as follows: after observing the noisy signal s_l, a player (or an outsider) with initial prior p_0 updates his or her prior to $P_{e,p_0}(\theta|s_l)$ (note it is independent of e); then the player observes a path of effort till time t e^t, an experimentation outcome history till t N^t, and he or she updates belief according to Bayes rule, using $P_{e,p_0}(\theta|s_l)$ the new "prior". This is how the informed player in our asymmetric information game updates his belief.

[56]That is, N^t refers to the constant function $0^{[0,t]}$.

Similarly, dividing both the numerator and denominator of the right-hand side of equation (27) by $\sum_{\tilde{\theta}} P_{e,p_0}(N^t|\tilde{\theta})P_{e,p_0}(\tilde{\theta})$, we have

$$
\begin{aligned}
P_{e,p_0}(\theta|s_l, N^t) &= \frac{P_{e,p_0}(s_l|\theta)[P_{e,p_0}(N^t|\theta)P_{e,p_0}(\theta)/\sum_{\tilde{\theta}} P_{e,p_0}(N^t|\tilde{\theta})P_{e,p_0}(\tilde{\theta})]}{\sum_{\theta} P_{e,p_0}(s_l|\theta)[P_{e,p_0}(N^t|\theta)P_{e,p_0}(\theta)/\sum_{\tilde{\theta}} P_{e,p_0}(N^t|\tilde{\theta})P_{e,p_0}(\tilde{\theta})]} \\
&= \frac{P_{e,p_0}(s_l|\theta)P_{e,p_0}(\theta|N^t)}{\sum_{\theta} P_{e,p_0}(s_l|\theta)P_{e,p_0}(\theta|N^t)}. \qquad (28)
\end{aligned}
$$

This equality can be interpreted as follows: after observing a path of effort till time t e^t, an experimentation outcome history till t N^t, a player (or an outsider) with initial prior p_0 updates his or her prior to $P_{e,p_0}(\theta|N^t)$; then the player observes the noisy signal s_l, and he or she updates belief according to Bayes rule, using $P_{e,p_0}(\theta|N^t)$ the new "prior".

(2) In the asymmetric information game, any public history (e^t, N^t) that leads to the same posterior of s_--type informed player must also lead to the same posterior of s_+-type informed player. This is because on the right-hand side of equation (28), $P_{e,p_0}(s_l|\theta)$ is independent of e and p_0, if the left-hand side when s_l replaced by s_- is equal to some p^-, then there is a unique value of $P_{e,p_0}(\theta|N^t)$ satisfying equation (28), denoted as p, and hence a unique value of the left-hand side of equation (28) when s_l replaced by s_+. That is, one variable, be it p^-, p^+, or p, is sufficient to represent the posteriors of the two types of the informed player. In this draft, we use p, that is, $P_{e,p_0}(\theta|N^t)$, and call it "background belief".

B Some Best Responses

B.1 U's best response to a full pooling Strategy

In the following Lemma, we assume both types of player I experiment with full resource over some interval of the back ground belief p, say $[\underline{p}, \bar{p}]$, in the asymmetric information game, and analyze U's best response. Given this full pooling strategy of I, the cumulative probability of revealing is a constant (over the interval of background belief). Let's denote it by Y. Then player I's reputation μ satisfies equation (8), and continuously decreases with time, according to equation (9) with dY_t being 0 (, for $p \in [\underline{p}, \bar{p}]$).

Lemma 5. *Assume over some interval $[\underline{p}, \bar{p}]$ both types of player I experiment with full resource (as a pure strategy). Let $\tilde{e}^U : [\underline{p}, \bar{p}] \times [0,1] \to [0,1]$ be U's best response to this strategy, and $\tilde{W}^U : [\underline{p}, \bar{p}] \times [0,1] \to \mathbb{R}$ her continuation value function if she plays the best response and player I plays the given strategy. Then*

1. *U finds it optimal to use corner solutions, that is, either to experiment with full resource, or not to experiment. At any point of p where U switches actions, her value satisfies $\tilde{W}^U(p, \boldsymbol{\mu}) = s + s - \lambda^U(p, \boldsymbol{\mu})h$.*

2. *If $\tilde{W}^U(p, \boldsymbol{\mu}) > s + s - \lambda^U(p, \boldsymbol{\mu})h$, then $e^U(p, \boldsymbol{\mu}) = 1$; If $\tilde{W}^U(p, \boldsymbol{\mu}) < s + s - \lambda^U(p, \boldsymbol{\mu})h$, then $e^U(p, \boldsymbol{\mu}) = 0$; If $\tilde{W}^U(p, \boldsymbol{\mu}) = s + s - \lambda^U(p, \boldsymbol{\mu})h$, then $e^U(p, \boldsymbol{\mu}) \in [0, 1]$.*

3. *If moreover U's value function satisfies the boundary condition* $W^U(\underline{p}, \boldsymbol{\mu}) = s + s - \lambda^U(\underline{p}, \boldsymbol{\mu})h$*, then she finds it optimal to adopt a cutoff strategy: to experiment with full resource if* $p \geq p^*$*, and not to experiment otherwise, for some* $p^* \in [\underline{p}, \bar{p}]$*.*

4. *If on top of the boundary condition in point 3, at* $p = \underline{p}$ *is also satisfied*

$$r\left(\lambda^U(p,\boldsymbol{\mu})h - s\right) + \lambda p(1-p)\frac{d\lambda^U(p,\boldsymbol{\mu})}{dp}h + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - s - \left(s - \lambda^U(p,\boldsymbol{\mu})h\right)\right) \geq 0 \quad (29)$$

then U finds it optimal to experiment with full resource over $[\underline{p}, \bar{p}]$*.*

Proof. Point 1. Given player I's strategy, player U's value function $\tilde{W}^U$ satisfies the following HJB equation, for $p \in (\underline{p}, \bar{p})$,

$$\begin{aligned} r\tilde{W}^U(p,\boldsymbol{\mu}) &= \max_{e\in[0,1]} e\left[r\left(\lambda^U(p,\boldsymbol{\mu})h - s\right) - \lambda p(1-p)\frac{d\tilde{W}^U(p,\boldsymbol{\mu})}{dp} + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p,\boldsymbol{\mu})\right)\right] \\ &\quad + \left[-\lambda p(1-p)\frac{d\tilde{W}^U(p,\boldsymbol{\mu})}{dp} + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p,\boldsymbol{\mu})\right)\right] + rs \end{aligned}$$

At any state $(p, \boldsymbol{\mu})$ where player U is indifferent between experimenting and not experimenting, we have

$$r\left(s - \lambda^U(p,\boldsymbol{\mu})h\right) = -\lambda p(1-p)\frac{d\tilde{W}^U(p,\boldsymbol{\mu})}{dp} + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p,\boldsymbol{\mu})\right) \quad (30)$$

and consequently the HJB equation of $\tilde{W}^U$ reduces to

$$r\tilde{W}^U(p,\boldsymbol{\mu}) - rs = -\lambda p(1-p)\frac{d\tilde{W}^U(p,\boldsymbol{\mu})}{dp} + \lambda^U(p,\boldsymbol{\mu})\left(\lambda h - \tilde{W}^U(p,\boldsymbol{\mu})\right)$$

The above equations implies that

$$\tilde{W}^U(p,\boldsymbol{\mu}) = s + s - \lambda^U(p,\boldsymbol{\mu}). \quad (31)$$

Since there is no subinterval of $[\underline{p}, \bar{p}]$ over which $\tilde{W}^U$ satisfy equation (30) and (31) simultaneously, there is no subinterval of $[\underline{p}, \bar{p}]$ over which player U strictly prefers an interior level of experimentation.

Point 2 is obvious with the above analysis.

Point 3. We will show that $\tilde{W}^U(p, \boldsymbol{\mu})$ intersects with $s + s - \lambda^U(p, \boldsymbol{\mu})$ at most once for $p \in (\underline{p}, \bar{p})$. Then combining *Point 2*, we obtain *Point 3*.

To show the former, it is sufficient to show that if there is some $\tilde{p} \in [\underline{p}, \bar{p})$ such that $\tilde{W}^U(\tilde{p}, \boldsymbol{\mu}) = s + s - \lambda^U(\tilde{p}, \boldsymbol{\mu})$, and $\frac{d\tilde{W}^U(\tilde{p}+,\boldsymbol{\mu}(\tilde{p}+))}{dp} > -\frac{d\lambda^U\left(\underline{p},\boldsymbol{\mu}(\underline{p})\right)}{dp}$, then $\tilde{W}^U(p, \boldsymbol{\mu}) > s + s - \lambda^U(p, \boldsymbol{\mu})$ for all $p \in (\tilde{p}, \bar{p})$. Suppose by negation there is some $\check{p} \in (\tilde{p}, \bar{p})$ such that $\tilde{W}^U(\check{p}, \boldsymbol{\mu}) =$

$s+s-\lambda^U(\check{p},\boldsymbol{\mu})$, then we must have $\frac{d\tilde{W}^U(\check{p}-,\boldsymbol{\mu}(\check{p}))}{dp} < \frac{d\tilde{W}^U(\tilde{p}+,\boldsymbol{\mu}(\tilde{p}+))}{dp}$, and $\tilde{W}^U(\check{p},\boldsymbol{\mu}) < \tilde{W}^U(\tilde{p},\boldsymbol{\mu})$ since $s+s-\lambda^U(p,\boldsymbol{\mu})$ strictly decreases in p. But these inequalities imply that equation (30) cannot be satisfied at both $\tilde{p}$ and $\check{p}$. A contradiction.

Point 4. Condition (29), together with equation (30) and $W^U(\underline{p},\boldsymbol{\mu}) = s+s-\lambda^U(\underline{p},\boldsymbol{\mu})$, implies that $\frac{d\tilde{W}^U\left(\underline{p}+,\boldsymbol{\mu}(\underline{p}+)\right)}{dp} > -\frac{d\lambda^U\left(\underline{p},\boldsymbol{\mu}(\underline{p})\right)}{dp}$, hence by the argument employed to prove *Point 3*, we conclude that $\tilde{e}^U(p,\boldsymbol{\mu}) = 1$ over $(\underline{p},\bar{p}]$.

□

B.2 Best response in the symmetric information game

In the symmetric information game, a counterpart of Lemma 5, with μ replaced by 0, is valid. More generally, if one player experiment with a constant level of resource over some interval of background belief, then the other player finds it optimal to use corner solutions over this interval, with at most two cutoffs. The proof is similar and hence omitted.

C Equilibrium Construction and Analysis

C.1 Beliefs

Lemma 6. *Let $\{p_t\}_{t\geq 0}$ denote the background belief process, $\{\mu_t\}_{t\geq 0}$ I's reputation process, and $\{Y_t\}_{t\geq 0}$ the process of cumulative probability of separation induced by a strategy profile (e^{I+}, e^{I-}, e^U) with e^{I+} being pure. If over an interval of time $[t_1,t_2]$ the revealing intensity corresponding to $\{Y_t\}_{t\geq 0}$ is finite and continuously differentiable with respect to time, and $\{(e_t^{I+}, e_t^U)\}_{t\geq 0}$ are continuous (over $[t_1,t_2]$), then the function $\tilde{\mu}$ defined by $\mu_t = \tilde{\mu}(p_t)$ for $t\in[t_1,t_2]$, is continuously differentiable in p over $[p_{t_2}, p_{t_1}]$. Moreover, over $[t_1,t_2]$, the reputation process $\{\mu_t\}_{t\geq 0}$ evolves according to*

$$d\mu_t \;=\; -\tilde{\mu}_p\left(p_t\right)\left(e_t^{I+}+e_t^U\right)p_t\left(1-p_t\right)\lambda_1 dt. \tag{32}$$

Proof. From equation (6), over any interval of time $[t_1,t_2]$ such that $(e_t^I + e_t^U) > 0$ and $p_t\in(0,1)$, we have $\{p_t\}_{t\geq 0}$ strictly decreases in t over $[t_1,t_2]$. Moreover, with $\{(e_t^{I+}, e_t^U)\}_{t\geq 0}$ continuous over $[t_1,t_2]$, we have $\frac{dt}{dp_t}$ finite and continuous for $t\in[t_1,t_2]$.

If the revealing intensity corresponding to $\{Y_t\}_{t\geq 0}$ is finite and continuous with respect to time for t in $[t_1,t_2]$, then $\{\frac{dY_t}{dp_t}\}_{t\geq 0}$ is also finite and continuously differentiable with respect to time for $t\in[t_1,t_2]$, using the chain rule $\frac{dY_t}{dp_t} = \frac{dY_t}{dt}\frac{dt}{dp_t}$ and the result $\{\frac{dt}{dp_t}\}_{t\geq 0}$ being finite and continuous for $t\in[t_1,t_2]$ obtained in the previous paragraph.

Applying equation (9), and that $p_t\in(0,1)$, we have $\frac{d\mu_t}{dp_t}$ is finite and continuous for $t\in[t_1,t_2]$. Therefore, $\tilde{\mu}$ is continuously differentiable over $[p_{t_2}, p_{t_1}]$, and satisfies

$$d\mu_t \;=\; \tilde{\mu}_p\left(p_t\right)dp_t,$$

which, together with equation (6), gives formula (32). □

C.2 Characterization of the gradual revelation phase

C.2.1 U's strategy (proof of the second case of Lemma 2)

We now derive player U's strategy in the gradual revelation phase for $p \in (p_1^{*-}, p_{gr})$ (if nonempty), assuming that the equilibrium with gradual revelation is an equilibrium.

Proof. First, $W^{I-}(p,0)$ satisfies the same HJB equation as equation (14), with the arguments $(p, \hat{\boldsymbol{\mu}}(p))$ in all functions replaced by $(p,0)$. Since for $p \in (p_1^{*-}, p_{gr})$, $e^{I-}(p,0) \in (0,1)$, the terms in equation (14) that are directly affected by type s_-'s effort must be 0:

$$\left[r\left(\lambda^{I-}(p)\,h - s\right) - \lambda p\,(1-p)\,\frac{dW^{I-}(p,0)}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}(p,0)\right)\right] = 0.$$

As $\lambda^{I-}(p)\,h - s < 0$ for $p \in (p_1^{*-}, p_{gr})$, we also have

$$\left[-\lambda p\,(1-p)\,\frac{dW^{I-}(p,0)}{dp} + \lambda^{I-}(p)\left(\lambda h - W^{I-}(p,0)\right)\right] > 0.$$

During gradual revelation phase, type s_- is indifferent between revealing and not revealing, and hence $W^{I-}(p, \hat{\boldsymbol{\mu}}(p)) = W^{I-}(p,0)$. This equality, together with both $W^{I-}(p, \hat{\boldsymbol{\mu}}(p))$ and $W^{I-}(p,0)$ satisfying the HJB equation (14), implies that $e^U(p, \hat{\boldsymbol{\mu}}(p)) = e^U(p,0)$ over (p_1^{*-}, p_{gr}). Since by construction $e^U(p,0) = e^{S-}(p)$, we have $e^U(p, \hat{\boldsymbol{\mu}}(p)) = e^{S-}(p)$. Therefore, the uninformed player's effort is decreasing over time during the non-rewarding region. □

C.2.2 U's HJB equation and experimentation incentive

We here offer a heuristic proof that U's value function during the gradual revelation phase satisfies equation (17).

Proof. Suppose at time-t state $(p_t, \hat{\boldsymbol{\mu}}(p_t))$, player U considers the following strategy: experimenting with resource $\tilde{e}$ during the time interval $[t, t+dt)$, experimenting with her equilibrium effort $e^U(p_t, \hat{\boldsymbol{\mu}}(p_t))$ during the time interval $[t+dt, t+2dt)$ if player I does not reveal over $[t, t+dt)$, and playing according to the candidate equilibrium strategy at other states.

The "required return" of doing so, $rW^U(p, \hat{\boldsymbol{\mu}}) \cdot 2dt$, should equal the right-hand side of equation (17): the expected current flow payoff in the $2dt$ duration of time,

$$r\left(\tilde{e} + e^U(p, \hat{\boldsymbol{\mu}})\right)\left(\lambda^U(p, \hat{\boldsymbol{\mu}})\,h - s\right)dt + 2rsdt,$$

plus the "capital gain",

$$E[W(p_{t+2dt}, \hat{\boldsymbol{\mu}}(p_{t+2dt})) - W(p_t, \hat{\boldsymbol{\mu}}(p_t))],$$

which can be further decomposed into three parts. The first two parts of the "capital gain" are similar with that in equation (14), that is, the change in her continuation value in case good news does not arrive and type s_- does not reveal his type in $[t, t+2dt)$, and the change in her continuation value in case good news arrives in $[t, t+2dt)$, multiplied by the probability of each event respectively. The third part of the "capital gain" comes from the possibility that type s_- might reveal his type in $[t, t+2dt)$, an event that would cause her continuation value to reduce by an amount $|W^U(p,0) - W^U(p, \hat{\boldsymbol{\mu}})|$.

It is crucial to notice that type s_-'s revealing intensity over $[t, t+dt)$, denoted as y, does not depend on player U's effort level $\tilde{e}$ over $[t, t+dt)$, while type s_-'s revealing intensity over $[t, t+2dt)$, denoted as $\tilde{y}$, does. We now analyze the latter effect. Conditional on type s_- does not reveal his type in the interval $[t, t+dt)$, the state at $t+dt$ before players move, will evolve to

$$\left(p_t - (1+\tilde{e})\,\lambda p\,(1-p)\,dt, \hat{\boldsymbol{\mu}}\,(p_t) - \hat{\boldsymbol{\mu}}_p\,(1+\tilde{e})\,\lambda p_t\,(1-p_t)\,dt\right),$$

which is below the curve $\hat{\boldsymbol{\mu}}$. Therefore, at time $t+dt$, according to the equilibrium prescription, type s_- will reveal with a probability such that the action of non-revealing will push the state up to the curve $\hat{\boldsymbol{\mu}}$ again, implying that the new state at $t+2dt$ will be

$$\left(p_t - \left(1+\tilde{e}+1+e^U\,(p_t, \hat{\boldsymbol{\mu}})\right)\lambda p_t\,(1-p_t)\,dt, \hat{\boldsymbol{\mu}}\left(p_t - \left(1+\tilde{e}+1+e^U\,(p_t, \hat{\boldsymbol{\mu}})\right)\lambda p_t\,(1-p_t)\,dt\right)\right).$$

Ignoring higher order terms of dt, the amount of adjustment in state variable in this $2dt$ duration of time equals to

$$\left(-\left(1+\tilde{e}+1+e^U\,(p_t, \hat{\boldsymbol{\mu}})\right)\lambda p_t\,(1-p_t)\,dt, -\hat{\boldsymbol{\mu}}_p\left(1+\tilde{e}+1+e^U\,(p_t, \hat{\boldsymbol{\mu}})\right)\lambda p_t\,(1-p_t)\,dt\right).$$

Employing equation (15), the amount of adjustment implies that in this $2dt$ duration of time, type s_- would reveal his type with probability

$$(y+\tilde{y})dt = \left(\phi\,(p,\mu) - \frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}\right)\left(1+\tilde{e}+1+e^U\,(p_t, \hat{\boldsymbol{\mu}})\right)\frac{p\,(1-p)\,\lambda}{(1-\hat{\boldsymbol{\mu}})}dt.$$

Collecting terms that share the same effort term together, we obtain

$$\begin{aligned} 2rW^U\,(p, \hat{\boldsymbol{\mu}}) &= \max_{\tilde{e}\in[0,1]} \tilde{e}\left[r\left(\lambda^U\,(p, \hat{\boldsymbol{\mu}})\,h - s\right) + A\,(p, \hat{\boldsymbol{\mu}})\right] + A(p,\hat{\mu}) + s \\ &\quad + e^U\,(p, \hat{\boldsymbol{\mu}})\left[r\left(\lambda^U\,(p, \hat{\boldsymbol{\mu}})\,h - s\right) + A(p,\hat{\mu})\right] + A\,(p, \hat{\boldsymbol{\mu}}) + s \end{aligned}$$

This equation is the same with equation (17), since $e^U(p,\hat{\mu})$ is a solution to the maximization problem.

During gradual revelation phase, U's equilibrium effort is interior (except at $p = p_1^{*-}$). Therefore, her IC condition (18) must hold.

This IC condition, together with the HJB equation (17), implies that player U's value function also satisfies equation (19). □

C.2.3 The gradual revelation path $\hat{\boldsymbol{\mu}}$ (proof of Lemma 3)

Formula of g.

Before introducing this ODE, we need to define two functions B and C:

$$\begin{aligned} B(p,\mu) &= \left(s-\lambda^U(p,\mu)h\right)-\left(s-\lambda^{I-}(p)h\right)e^{I-}(p,0), \\ C(p,\mu) &= r\left(\lambda^U(p,\mu)h-s\right)+\lambda p(1-p)\lambda_p^U(p,\mu)h+\lambda^U(p,\mu)\left(\lambda h-s-\left(s-\lambda^U(p,\mu)h\right)\right), \end{aligned}$$

where B is player U's value reduction caused by type s_-'s revealing action, and C could be interpreted as player U's expected pseudo-payoff of experimentation had μ being a constant. Then, using U's indifference condition (18) and her value function (19), we have

$$\begin{aligned} \hat{\boldsymbol{\mu}}_p &= g(p,\hat{\boldsymbol{\mu}}) \\ &\equiv -\left(\frac{C(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}-\phi(p,\hat{\boldsymbol{\mu}})\frac{B(p,\hat{\boldsymbol{\mu}})}{B(p,0)}\right)\hat{\boldsymbol{\mu}}, \quad p\in\left(p_2^{*-},p_{gr}\right) \end{aligned} \tag{33}$$

Before proving Lemma 3, we derive some preliminary results. We first derive some convenient formula for for $\hat{\boldsymbol{\mu}}_p/\hat{\boldsymbol{\mu}}$ and for $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp$, which will be used in this section and the following sections. Then In Lemma 7, we show that $\hat{\boldsymbol{\mu}}$ is a strictly decreasing function. With this lemma, we show that the ODE problem defined by equations (20)-(22) has a unique solution, in Lemma 8.

We now establish equalities (34) to (37):

$$\begin{aligned} -\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}} &= \frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h/dp}{B(p,\hat{\boldsymbol{\mu}})}+\frac{1}{\lambda p(1-p)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right. \\ &\quad \left. +\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I+}(p)\right] \end{aligned} \tag{34}$$

$$\begin{aligned} -\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp} &= \frac{\lambda_\mu^U\hat{\boldsymbol{\mu}}B(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right. \\ &\quad \left. +\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda+\lambda^{I-}(p)-\frac{1}{\hat{\boldsymbol{\mu}}}\left(\frac{\rho_g(1-\rho_b)}{\rho_b-\rho_g}\lambda+\lambda^{I-}(p)\right)\right] \qquad (35) \\ &= \frac{\lambda_\mu^U\hat{\boldsymbol{\mu}}B(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right. \\ &\quad \left. +\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda+\lambda^{I-}(p)-\frac{\left(\lambda-\lambda^{I-}(p)\right)\lambda^{I-}(p)}{\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I-}(p)}\right] \qquad (36) \\ &= \frac{\lambda_\mu^U\hat{\boldsymbol{\mu}}B(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right. \\ &\quad \left. -\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda-\lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I-}(p)}\right]. \qquad (37) \end{aligned}$$

Recall the definitions of $\boldsymbol{\mu}^o$, λ^{I+}, λ^{I-}, λ^U, and ϕ (page 12, 29), we have

$$\begin{aligned}
\phi(p,\mu) &\equiv \frac{(1-\mu)(\rho_b-\rho_g)}{\boldsymbol{\mu}^o(p)(1-\boldsymbol{\mu}^o(p))} \\
&= \frac{(1-\mu)\left(\lambda^{I+}(p)-\lambda^{I-}(p)\right)}{\lambda p(1-p)} \\
&= \frac{\lambda^{I+}(p)-\lambda^U(p,\mu)}{\lambda p(1-p)},
\end{aligned} \tag{38}$$

Combining equations (20) and (38), at p such that $\hat{\boldsymbol{\mu}}(p)\neq 0$, we have

$$\begin{aligned}
-\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}} &= \frac{C(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)} - \phi(p,\hat{\boldsymbol{\mu}})\frac{B(p,\hat{\boldsymbol{\mu}})}{B(p,0)} \\
&= \frac{\lambda_p^U(p,\hat{\boldsymbol{\mu}})h}{B(p,0)} + \frac{B(p,\hat{\boldsymbol{\mu}})}{\lambda p(1-p)B(p,0)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}\right. \\
&\quad \left. +\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I+}(p)\right]
\end{aligned} \tag{39}$$

Applying equalities $d\lambda^U/dp = \lambda_p^U + \lambda_\mu^U\hat{\boldsymbol{\mu}}_p$ and $B(p,0) = B(p,\hat{\boldsymbol{\mu}}) + \lambda_\mu^U\hat{\boldsymbol{\mu}}h$, we arrive at equation (34).

We now derive an explicit formula of $d\lambda^U(p,\hat{\boldsymbol{\mu}})h/dp$. Before this, we need an explicit form of $\frac{\lambda_p^U}{\lambda_\mu^U\hat{\boldsymbol{\mu}}}$.

By the definition of $\lambda^{I+}(p)$ and $\lambda^{I-}(p)$, we have

$$\lambda^{I+}(p)-\lambda^{I-}(p) = \frac{\lambda p(1-p)\left(\frac{\rho_b}{\rho_g}-\frac{1-\rho_b}{1-\rho_g}\right)}{\left(p+(1-p)\frac{1-\rho_b}{1-\rho_g}\right)\left(p+(1-p)\frac{\rho_b}{\rho_g}\right)}$$

Expanding λ_p^U, λ_μ^U, and apply the above equation, we have

$$\begin{aligned}
\frac{\lambda_p^U}{\lambda_\mu^U\hat{\boldsymbol{\mu}}} &= \frac{\lambda_p^U}{\hat{\boldsymbol{\mu}}(\lambda^{I+}(p)-\lambda^{I-}(p))} \\
&= \frac{\rho_g(1-\rho_g)}{p(1-p)(\rho_b-\rho_g)}\left[\frac{p+(1-p)\frac{\rho_b}{\rho_g}}{p+(1-p)\frac{1-\rho_b}{1-\rho_g}}\frac{1-\rho_b}{1-\rho_g}+\left(\frac{1}{\hat{\boldsymbol{\mu}}}-1\right)\frac{p+(1-p)\frac{1-\rho_b}{1-\rho_g}}{p+(1-p)\frac{\rho_b}{\rho_g}}\frac{\rho_b}{\rho_g}\right] \\
&= \frac{\lambda}{\lambda p(1-p)}\left[\left(\frac{\rho_b(1-\rho_g)}{\rho_b-\rho_g}-\frac{\lambda^{I+}(p)}{\lambda}\right)+\left(\frac{1}{\hat{\boldsymbol{\mu}}}-1\right)\left(\frac{\rho_g(1-\rho_b)}{\rho_b-\rho_g}+\frac{\lambda^{I-}(p)}{\lambda}\right)\right] \\
&= \frac{\lambda}{\lambda p(1-p)}\left[\left(1-\frac{\lambda^{I+}(p)}{\lambda}\right)+\frac{1}{\hat{\boldsymbol{\mu}}}\frac{\rho_g(1-\rho_b)}{\rho_b-\rho_g}+\left(\frac{1}{\hat{\boldsymbol{\mu}}}-1\right)\frac{\lambda^{I-}(p)}{\lambda}\right]
\end{aligned} \tag{40}$$

Subtracting $\frac{\lambda_p^U}{\lambda_\mu^U \hat{\boldsymbol{\mu}}} + \frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h/dp}{B(p,\hat{\boldsymbol{\mu}})}$ from both sides of equation (34), and using equation (40), we have

$$\begin{aligned}
& -\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp}\frac{B(p,0)}{\lambda_\mu^U\hat{\boldsymbol{\mu}}B(p,\hat{\boldsymbol{\mu}})} \\
= & \frac{1}{\lambda p(1-p)}\left[\frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}+\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I+}(p)\right. \\
& \left.-\lambda+\lambda^{I+}(p)+\lambda^{I-}(p)-\frac{1}{\hat{\boldsymbol{\mu}}}\left(\frac{\rho_g(1-\rho_b)}{\rho_b-\rho_g}\lambda+\lambda^{I-}(p)\right)\right]
\end{aligned}$$

Equation (35) follows from this equation immediately. Using the following equality, which is obtained after some algebra,

$$\frac{1}{\hat{\boldsymbol{\mu}}}\left(\frac{\rho_g(1-\rho_b)}{\rho_b-\rho_g}\lambda+\lambda^{I-}(p)\right)=\frac{\left(\lambda-\lambda^{I-}(p)\right)\lambda^{I-}(p)}{\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda^{I-}(p)},$$

we obtain equality (36). Rearranging terms, we have equality (37).

Before proving existence of solution to the ODE problem (20)-(22), we will show that $\hat{\boldsymbol{\mu}}$ satisfying equation (20) has some a priori bound (if we impose some conditions that are necessary for the candidate equilibrium to be an equilibrium). Lemma 7 is a useful step towards this. Also, note that $B(p^{S-},0)=0$ (since $e^{S-}(p^{S-})=1$), and $B(p,0)>0$ for $p\in[p_2^{*-},p^{S-})$, hence we will treat the point p^{S-} with care.

Lemma 7. *Let $\alpha\in(p_2^{*-},p^{S-})$ and $\hat{\boldsymbol{\mu}}|_{[p_2^{*-},\alpha)}$ be a solution to the ODE problem defined by (20) restricted over $[p_2^{*-},\alpha]$ and the initial condition (21). If $\hat{\boldsymbol{\mu}}\in(0,1)$ and $B(p,\hat{\boldsymbol{\mu}})>0$ over (p_2^{*-},α), then $\hat{\boldsymbol{\mu}}_p<0$ over (p_2^{*-},α).*

Proof. Define

$$D(p,\hat{\boldsymbol{\mu}}) \equiv \frac{r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)}{B(p,\hat{\boldsymbol{\mu}})}+\lambda^U(p,\hat{\boldsymbol{\mu}})-\lambda. \tag{41}$$

Note that $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp\le 0$, implies $\hat{\boldsymbol{\mu}}_p<0$. Therefore, if $D(p,\hat{\boldsymbol{\mu}})>0$ on a Gradual Revelation path, then Lemma 7 would follow. We now show $D(p,\hat{\boldsymbol{\mu}})>0$.

Observe that, (i) if $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp\le 0$, then $D(p,\hat{\boldsymbol{\mu}})>0$, from equality (36) and the definition of D; (ii) if $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp>0$, then $B(p,\hat{\boldsymbol{\mu}}(p))$ strictly decreases as p increases, and hence $D(p,\hat{\boldsymbol{\mu}})$ strictly increases as p increases. Therefore, if we show $D(p,\hat{\boldsymbol{\mu}})>0$ at $p=p_2^{*-}$, then by continuity of D and the above two observations, we have $D(p,\hat{\boldsymbol{\mu}})>0$ for $p\in[p_2^{*-},p_{gr})$.

From now till the end of this proof, if not mentioned, p is fixed at p_2^{*-}. Using the initial condition (21), at $p=p_2^{*-}$, we have

$$\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})(\lambda h-s)}{s-\lambda^U(p,\hat{\boldsymbol{\mu}})h}=\frac{s\left(\lambda^{I+}(p)-\lambda^{I-}(p)\right)+\lambda^{I-}(p)\left(W^{S+}(p)-s\right)}{(s-\lambda^{I-}(p)h)(W^{S+}(p)-s)}(\lambda h-s)$$

Applying the definition of D, and the fact that $e^{S-}(p) = 0$ at $p = p_2^{*-}$, we obtain

$$D\left(p, \hat{\boldsymbol{\mu}}\right) + \lambda - \lambda^{I+}(p) = \frac{1}{s - \lambda^{I-}(p)\, h}\left[s\left(\lambda^{I+}(p) - \lambda^{I-}(p)\right)\frac{\lambda h - W^{S+}(p)}{W^{S+}(p) - s} + \lambda^{I-}(p)\left(\lambda^{I+}(p) - \lambda h\right)\right]$$

Employing the definition of p_2^{*-} (equation (1)) and of λ^{I-}, the above equality becomes

$$D\left(p, \hat{\boldsymbol{\mu}}\right) + \lambda - \lambda^{I+}(p) = \frac{-W^{S+}(p)\left(r + 2\lambda^{I+}(p)\right) + (2\lambda + r)\,\lambda^{I+}(p)\, h}{2\left(W^{S+}(p) - s\right)} \tag{42}$$

By Assumption 1, $p_2^{*-} \geq p^{S+}$, we have $e^{S+} = 1$ at $p = p_2^{*-}$. Recall w^S is the continuation value function corresponding to the symmetric MPE under symmetric information, which is a function of the *true posterior*, rather than the *background belief*, that is, $w^S(\boldsymbol{q}^+(p)) = W^{S+}(p)$ (with $\boldsymbol{q}^+$ defined in equation (3)).

Since in the symmetric MPE in the symmetric information setup, both players experiment with full resource if their common posterior is above q^S, w^S satisfies the following HJB equation for $\boldsymbol{q}^+(p) > q^S$,

$$rw^S\left(\boldsymbol{q}^+\right) - rs = r\left(\lambda\boldsymbol{q}^+h - s\right) - 2\lambda\boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right)w_q^S\left(\boldsymbol{q}^+\right) + 2\lambda\boldsymbol{q}^+\left(\lambda h - w^S\left(\boldsymbol{q}^+\right)\right),$$

where the argument of $\boldsymbol{q}^+$ is omitted.

Rearranging terms, we have

$$-w^S\left(\boldsymbol{q}^+\right)\left(r + 2\lambda\boldsymbol{q}^+\right) + (2\lambda + r)\,\lambda\boldsymbol{q}^+h = 2\lambda\boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right)w_q^S\left(\boldsymbol{q}^+\right). \tag{43}$$

With this equation, equation (42) becomes

$$D\left(p, \hat{\boldsymbol{\mu}}\right) + \lambda - \lambda^{I+}(p) = \frac{\lambda\boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right)w_q^S\left(\boldsymbol{q}^+\right)}{\left(w^S\left(\boldsymbol{q}^+\right) - s\right)}. \tag{44}$$

Therefore,

$$\begin{aligned}
D\left(p, \hat{\boldsymbol{\mu}}\right) &= \frac{\lambda\boldsymbol{q}^+\left(1 - \boldsymbol{q}^+\right)w_q^S\left(\boldsymbol{q}^+\right)}{\left(w^S\left(\boldsymbol{q}^+\right) - s\right)} - \left(1 - \boldsymbol{q}^+\right)\lambda \\
&= \left(1 - \boldsymbol{q}^+\right)\lambda\frac{\boldsymbol{q}^+w_q^S\left(\boldsymbol{q}^+\right) - \left(w^S\left(\boldsymbol{q}^+\right) - s\right)}{\left(w^S\left(\boldsymbol{q}^+\right) - s\right)} && (45) \\
&> \left(1 - \boldsymbol{q}^+\right)\lambda\frac{\left(\boldsymbol{q}^+ - q_1^*\right)w_q^S\left(\boldsymbol{q}^+\right) - \left(w^S\left(\boldsymbol{q}^+\right) - s\right)}{\left(w^S\left(\boldsymbol{q}^+\right) - s\right)} \\
&> 0 && (46)
\end{aligned}$$

The second-to-last inequality is due to $w_q^S(\boldsymbol{q}^+) > 0$ at $p = p_2^{*-}$; the last inequality is due to the convexity of w^S over $[q_1^*, 1]$, and that $w^S\left(q_1^*\right) = s$. □

Lemma 8. *If $\hat{\boldsymbol{\mu}}(p_2^{*-})$ defined by* (21) *is such that $\hat{\boldsymbol{\mu}}(p_2^{*-}) < \boldsymbol{\mu}^o(p_2^{*-})$, then the ODE problem defined by* (20)-(22) *has a unique solution $\hat{\boldsymbol{\mu}}$. Moreover, the right boundary p_{gr} is in (p_2^{*-}, p^{S-}).*

Proof. Let $\epsilon \in (0, p^{S-})$ be such that

$$s + s - \lambda^U \left(p^{S-} - \epsilon, \boldsymbol{\mu}^o \left(p^{S-} - \epsilon\right)\right) h = W^{S-} \left(p^{S-} - \epsilon\right).$$

Such an ϵ exists because both sides of the above equation are continuous in ϵ, the left-hand side is strictly greater than the right-hand side at $\epsilon = p^{S-}$, and strictly smaller than the latter at $\epsilon = 0$. As $\epsilon > 0$, we have $e^{I-}(p, 0) < 1$ over $[p_2^{*-}, p^{S-} - \epsilon]$,[57] and hence $B(p, 0)$ is bounded for $p \in [p_2^{*-}, p^{S-} - \epsilon]$. $C(p, \hat{\boldsymbol{\mu}})$, $\phi(p, \hat{\boldsymbol{\mu}})$, and $B(p, \hat{\boldsymbol{\mu}})$ are also bounded for $p \in [p_2^{*-}, p^{S-} - \epsilon]$ and $\hat{\boldsymbol{\mu}} \in [0, 1]$. To restrict $\hat{\boldsymbol{\mu}}$ to take values in $[0, 1]$, define

$$\chi(\mu) = \begin{cases} \mu, & \text{if } \mu \in [0, 1]; \\ 0, & \text{if } \mu < 0; \\ 1, & \text{if } \mu > 1. \end{cases}$$

Existence. Consider the following initial value problem

$$\hat{\boldsymbol{\mu}}_p = -\left(\frac{C\left(p, \chi\left(\hat{\boldsymbol{\mu}}\right)\right)}{\lambda p\left(1-p\right) B\left(p, 0\right)} - \phi\left(p, \chi\left(\hat{\boldsymbol{\mu}}\right)\right) \frac{B\left(p, \chi\left(\hat{\boldsymbol{\mu}}\right)\right)}{B\left(p, 0\right)}\right) \chi\left(\hat{\boldsymbol{\mu}}\right), \qquad p \in (p_2^{*-}, p^{S-} - \epsilon), \quad (47)$$

with the initial condition (21). $\hat{\boldsymbol{\mu}}_p$ as a function of $(p, \hat{\boldsymbol{\mu}})$, defined by equation (47), is bounded, and Lipschitz continuous. According to standard theorems (for example, Picard–Lindelöf theorem), this initial value problem has a unique solution, denoted as as $\hat{\boldsymbol{\mu}}$ (with an abuse of notation). Let $p_{gr} \equiv \inf\{p \in [p_2^{*-}, p^{S-} - \epsilon] \mid \hat{\boldsymbol{\mu}}(p) > \boldsymbol{\mu}^o(p)\}$.

We will show at the end of this proof that $\hat{\boldsymbol{\mu}}$ satisfies

Claim 2. *Over the interval (p_2^{*-}, p_{gr}), we have $\hat{\boldsymbol{\mu}} \in (0, 1)$, and $B(p, \hat{\boldsymbol{\mu}}) > 0$.*

Claim 3. $\hat{\boldsymbol{\mu}}\left(p^{S-} - \epsilon\right) < \boldsymbol{\mu}^o\left(p^{S-} - \epsilon\right)$.

Claim 2 says that $\hat{\boldsymbol{\mu}}$, the unique solution to ODE (47) and (21), satisfies ODE (20) over $[p_2^{*-}, p_{gr}]$. Claim 3, together with $\hat{\boldsymbol{\mu}}\left(p_2^{*-}\right) > \boldsymbol{\mu}^o\left(p_2^{*-}\right)$ and the continuity of $\hat{\boldsymbol{\mu}}$ and $\boldsymbol{\mu}^o$, implies that $p_{gr} \in (p_2^{*-}, p^{S-} - \epsilon)$.

Therefore, $\hat{\boldsymbol{\mu}}$ restricted over $[p_2^{*-}, p_{gr}]$ is a solution to the ODE problem (20)-(22).

Uniqueness. $\hat{\boldsymbol{\mu}}$ restricted over $[p_2^{*-}, p_{gr}]$ is the unique solution to this ODE problem. Suppose this ODE problem has another solution $\check{\boldsymbol{\mu}}$, and let $\check{p} \in [p_2^{*-}, p_{gr}]$ be the infimum of p such that $\check{\boldsymbol{\mu}}$ differs from $\hat{\boldsymbol{\mu}}$. Then we have $\check{\boldsymbol{\mu}}(\check{p}) = \hat{\boldsymbol{\mu}}(\check{p}) \in (0, 1)$, and $\check{\boldsymbol{\mu}} \in (0, 1)$ over $[\check{p}, \check{p} + \eta]$ for some small η. Hence $\check{\boldsymbol{\mu}}|_{[}\check{p}, \check{p} + \eta]$ is a solution to the ODE problem (47) restricted over $[\check{p}, \check{p} + \eta]$ with the initial value given by $\check{\boldsymbol{\mu}}(\check{p}) = \hat{\boldsymbol{\mu}}(\check{p})$, which contradicted with the latter having a unique solution.

We now prove the two claims above.

Proof of Claim 3. Suppose $\hat{\boldsymbol{\mu}}\left(p^{S-} - \epsilon\right) \geq \boldsymbol{\mu}^o\left(p^{S-} - \epsilon\right)$, then $W^U(p^{S-} - \epsilon, \hat{\boldsymbol{\mu}}(p^{S-} - \epsilon))$ defined by equation (19) is smaller than $W^{S-}(p^{S-} - \epsilon)$ by our choice of ϵ, then using the equation that $B(p, \hat{\boldsymbol{\mu}}) = W^U(p, \hat{\boldsymbol{\mu}}) - W^{S-}(p)$, we have $B(p^{S-} - \epsilon, \hat{\boldsymbol{\mu}}(p^{S-} - \epsilon)) \leq 0$. This contradicts with Claim 2. □

[57] Recall that over $[p_2^{*-}, p^{S-}]$, $e^{I-}(p, 0) \equiv e^{S-}(p)$, and is strictly increasing from 0 to 1.

Proof of Claim 2. Suppose by negation that there is some $p \in [p_2^{*-}, p_{gr}]$ such that $B(p, \hat{\boldsymbol{\mu}}) \leq 0$; denote the smallest p satisfying this inequality as $\tilde{p}$. Since $B(p, \hat{\boldsymbol{\mu}}) > 0$ at $p = p_2^{*-}$ and B is continuous, we have $\tilde{p} > p_2^{*-}$, and $B(p, \hat{\boldsymbol{\mu}}) > 0$ for $p \in [p_2^{*-}, \tilde{p})$.

$\hat{\boldsymbol{\mu}}$ is strictly decreasing over $[p_2^{*-}, \tilde{p})$. Because otherwise, there would exist a $\tilde{\tilde{p}} \in (p_2^{*-}, \tilde{p})$ such that $\hat{\boldsymbol{\mu}}_p(\tilde{\tilde{p}}) = 0$,[58] implying that $\hat{\boldsymbol{\mu}}|_{[p_2^{*-}, \tilde{\tilde{p}})}$ satisfies ODE (20) when restricted over $[p_2^{*-}, \tilde{\tilde{p}})$, and the initial condition (21), and yet it violates Lemma 7, a contradiction. Therefore, $\chi(\hat{\boldsymbol{\mu}}) = \hat{\boldsymbol{\mu}}$ for $p \in [p_2^{*-}, \tilde{p}]$, hence $\hat{\boldsymbol{\mu}}$ also satisfies ODE (20). But the function W^U defined by $s + s - \lambda^U(\hat{\boldsymbol{\mu}}, p)h$ (that is, equation (19)) would not satisfy equation (18) (because the left-hand side>0 for p sufficiently close to $\tilde{p}$), which contradicts with $\hat{\boldsymbol{\mu}}$ satisfying ODE (20). □

□

Proof of Lemma 3. First, $\hat{\boldsymbol{\mu}}$ is continuous over (p_2^{*-}, p_{gr}). Suppose by negation that there is some $\tilde{p} \in (p_2^{*-}, p_{gr})$ at which $\hat{\boldsymbol{\mu}}$ is discontinuous, that is, $\hat{\boldsymbol{\mu}}(\tilde{p}+) < \hat{\boldsymbol{\mu}}(\tilde{p}-)$.[59] Since over a small right neighborhood of $\tilde{p}$, player U is indifferent between experimenting and not experimenting, we must have $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+)) = s + s - \lambda^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))h$, by equation (19). Similarly, we have $W^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-)) = s + s - \lambda^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))h$. Since λ^U strictly increases in its second argument, these two inequalities imply that $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+)) > W^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))$. But this contradicts with the fact that $W^U \geq s$, as $W^U(p-, \hat{\boldsymbol{\mu}}(\tilde{p}-))$ is the average between $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))$ and s.

Similarly, $\hat{\boldsymbol{\mu}}$ is continuous at p_{gr}. The difference between this case and the previous case is that, over a small right neighborhood of p_{gr}, player U strictly prefers to experiment, and type s_- strictly prefers not to reveal. Therefore, by Point 2 of Lemma 5, we have $W^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+)) \geq s + s - \lambda^U(p+, \hat{\boldsymbol{\mu}}(\tilde{p}+))h$. Continuity of $\hat{\boldsymbol{\mu}}$ at p_{gr} follows the same logic as in the previous case.

Finally, we show that $\hat{\boldsymbol{\mu}}$ does not have singular continuous part. Let $\tilde{\boldsymbol{\mu}}$ be the solution to the ODE problem (20)-(22), and let $\tilde{W}^U$ be player U's continuation value function corresponding to the equilibrium associated with Gradual Revelation path $\tilde{\boldsymbol{\mu}}$.[60] Suppose by negation there is another Gradual Revelation path $\check{\boldsymbol{\mu}}$ (corresponding to another equilibrium which takes the same feature with the candidate equilibrium) that satisfies equation (20) almost everywhere, equations (21), and (22), and that $\tilde{\boldsymbol{\mu}}(p')$ differs from $\check{\boldsymbol{\mu}}(p')$ for some $p' \in (p_2^{*-}, p_{gr})$. Without loss of generality, suppose $\tilde{\boldsymbol{\mu}}(p') > \check{\boldsymbol{\mu}}(p')$. Let p'' be the largest p's such that $p \leq p'$ and that $\tilde{\boldsymbol{\mu}}(p) \geq \check{\boldsymbol{\mu}}(p)$. Existence of p'' is due to the continuity of $\tilde{\boldsymbol{\mu}}$ and $\check{\boldsymbol{\mu}}$, and that $\tilde{\boldsymbol{\mu}}(p_2^{*-}) = \check{\boldsymbol{\mu}}(p_2^{*-})$ (from the initial condition (21)). By the definition of p'', we have $\limsup_{\epsilon \downarrow 0} \frac{\check{\boldsymbol{\mu}}(p''+\epsilon) - \check{\boldsymbol{\mu}}(p'')}{\epsilon} < \tilde{\boldsymbol{\mu}}_p(p'')$. Let $\check{W}^U$ be U's continuation value function of the equilibrium with Gradual Revelation path $\check{\boldsymbol{\mu}}$. Then $\tilde{\boldsymbol{\mu}}(p) > \check{\boldsymbol{\mu}}(p)$ over (p'', p') implies that $\check{W}^U(p, \check{\boldsymbol{\mu}}) > \tilde{W}^U(p, \tilde{\boldsymbol{\mu}})$. We now show that $\tilde{\boldsymbol{\mu}}(p) > \check{\boldsymbol{\mu}}(p)$ over (p'', p') and that $\tilde{\boldsymbol{\mu}}(p'') = \check{\boldsymbol{\mu}}(p'')$ imply that $\check{W}^U(p, \check{\boldsymbol{\mu}}) < \tilde{W}^U(p, \tilde{\boldsymbol{\mu}})$ for $p \in (p'', p'' + \epsilon_1)$, if ϵ_1 small enough. A contradiction.

---

[58] $\tilde{\tilde{p}} > p_2^{*-}$ because $\hat{\boldsymbol{\mu}}(p_2^{*-}) < 0$.

[59] Note in our candidate equilibrium, $\hat{\boldsymbol{\mu}}$ is discontinuous if and only if type s_- reveals with a lump-sum probability, hence at any p, $\hat{\boldsymbol{\mu}}$ can only jump downward.

[60] Need result from the Verification section showing that this is indeed an equilibrium.

Fix a small $\epsilon > 0$, we change the strategies of type s_- and of player U in the equilibrium associated with Gradual Revelation path $\check{\boldsymbol{\mu}}$ as follows: starting at $(p'' + \epsilon, \check{\boldsymbol{\mu}}(p'' + \epsilon))$, type s_- does not reveal his type as long as background belief is in $(p'', p'' + \epsilon]$; at the background belief p'', he reveals with a lump-sum probability such that his reputation jumps to $\check{\boldsymbol{\mu}}(p'')$; at all other states, he plays his equilibrium strategy associated with Gradual Revelation path $\check{\boldsymbol{\mu}}$. U plays a best response to I's new strategy (, which is 0 effort). Denote U's continuation value corresponding to this new strategy profile as $\check{\check{W}}^U$, which will be written as simply a function of the background p, for ease of notation. As type s_- works harder in this new strategy profile, we have $\check{\check{W}}^U \geq \check{W}^U$ at $p = p'' + \epsilon$.[61] At $p = p''$, according to the new prescription, type s_- will reveal with probability $\frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)}$,[62] where $\check{Y}$ solves

$$\check{\boldsymbol{\mu}}(p'' + \eta) = \frac{\boldsymbol{\mu}^o(p'' + \eta)}{\boldsymbol{\mu}^o(p'' + \eta) + [1 - \boldsymbol{\mu}^o(p'' + \eta)]\left(1 - \check{Y}(\eta)\right)}, \tag{48}$$

for $\eta = 0, \epsilon$. That is, $\check{Y}(\epsilon)$ is type s_-'s cumulative revelation probability giving him a reputation $\check{\boldsymbol{\mu}}(p'' + \epsilon)$, when the background belief is $p'' + \epsilon$. Similarly we can define $\check{Y}(0)$, $\tilde{Y}(\epsilon)$, and $\tilde{Y}(0)$.

Since according to the new strategy of type s_-, he will reveal with a lump-sum probability at p'', we have

$$\check{\check{W}}^U(p''+) = \frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)} s + \left(1 - \frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)}\right)\check{W}^U(p'', \check{\boldsymbol{\mu}}(p'')). \tag{49}$$

Let t_ϵ denote the time it takes for the background belief to drop from $p'' + \epsilon$ to p''. By using equation (49), and the fact that U's best response to I's pooling strategy over $(p'', p'' + \epsilon)$ is 0 effort, we have

$$\check{\check{W}}^U(p'' + \epsilon) = (rs + \lambda^U \lambda h)t_\epsilon + (1 - rt_\epsilon - \lambda^U t_\epsilon)\left(\frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)} s + \left(1 - \frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)}\right)\check{W}^U(p'')\right) \tag{50}$$

$(rs + \lambda^U \lambda h)t_\epsilon$ is the "flow" value of both players' effort to player U in the t_ϵ duration of time: as U does not experiment, she receives rst_ϵ from her own arm; player I experiments with full resource, hence to player U, good news will arrive with probability $\lambda^U t_\epsilon$, leading to a discounted value λh.

Now coming back to the equilibrium with Gradual Revelation Path $\tilde{\boldsymbol{\mu}}$. Since U is indifferent between experimenting and not experimenting as long as no revealing and $p \in (p'', p'' + \epsilon)$, not experimenting is an optimal strategy for U. Hence we have

$$\tilde{W}^U(p'' + \epsilon) = (rs + \lambda^U \lambda h)t_\epsilon + (1 - rt_\epsilon - \lambda^U t_\epsilon)\left(\frac{\tilde{Y}(0) - \tilde{Y}(\epsilon)}{1 - \tilde{Y}(\epsilon)} s + \left(1 - \frac{\tilde{Y}(0) - \tilde{Y}(\epsilon)}{1 - \tilde{Y}(\epsilon)}\right)\tilde{W}^U(p'') + o(t_\epsilon)\right) \tag{51}$$

[61] That is, $\check{\check{W}}^U(p'' + \epsilon) \geq \check{W}^U(p'' + \epsilon, \check{\boldsymbol{\mu}}(p'' + \epsilon))$.

[62] Note with type s_-'s new revealing strategy, $\check{Y}(\epsilon)$ will be a constant when background belief is in $(p'', p'' + \epsilon]$.

where we use the fact that during t_ϵ duration of time, type s_-'s expected effort differs from 1 with probability $\frac{\tilde{Y}(0)-\tilde{Y}(\epsilon)}{1-\tilde{Y}(\epsilon)}$, which is of order t_ϵ (since by assumption $\tilde{\boldsymbol{\mu}}$ is absolutely continuous), causing a difference in U's flow payoff during this t_ϵ time of second order of t_ϵ.

Since $\tilde{\boldsymbol{\mu}}(p) > \check{\boldsymbol{\mu}}(p)$ over (p'', p'), and $\tilde{\boldsymbol{\mu}}(p'') = \check{\boldsymbol{\mu}}(p'')$, we have $\tilde{Y}(0) = \check{Y}(0)$, and $\tilde{Y}(\epsilon) > \check{Y}(\epsilon)$, hence

$$\frac{\tilde{Y}(0) - \tilde{Y}(\epsilon)}{1 - \tilde{Y}(\epsilon)} < \frac{\check{Y}(0) - \check{Y}(\epsilon)}{1 - \check{Y}(\epsilon)}.$$

Recall we also have $\check{W}^U(p, \check{\boldsymbol{\mu}}) = \tilde{W}^U(p, \tilde{\boldsymbol{\mu}})$ at $p = p''$. Therefore, by equations (50) and (51), we have $\check{\tilde{W}}^U(p'' + \epsilon) < \tilde{W}^U(p'' + \epsilon)$. As $\check{\tilde{W}}^U(p'' + \epsilon) \geq \check{W}^U(p'' + \epsilon)$, we also have $\check{W}^U(p'' + \epsilon) < \tilde{W}^U(p'' + \epsilon)$.

Since this inequality holds for all small $\epsilon > 0$, we conclude that $\check{W}^U(p, \check{\boldsymbol{\mu}}) < \tilde{W}^U(p, \tilde{\boldsymbol{\mu}})$ for $p \in (p'', p'' + \epsilon_1)$, if ϵ_1 small enough.

□

C.3 Verification (proof of Lemma 4)

As the argument for type s_+'s incentive compatibility only depends on whether the stage is full separation or not, we first analyze his incentive. We then check type s_-'s and player U's incentive stage by stage, backwardly.

At the full separation region ($p \leq p_2^{*-}$), if the state is $(p, 1)$, type s_+ has no incentive to deviate because both him and the uninformed player play the symmetric MPE with public information s_+. If the state is $(p, 0)$, then the uninformed player does not experiment, hence it is optimal for type s_+ to adopt the individually optimal solution.

Before full separation ($p > p_2^{*-}$), type s_+'s incentive to follow the prescribed equilibrium strategy is implied by the following lemma, since in the prescribed equilibrium, there is no $p > p_1^{*-}$ such that $e^{I-}(p, 0) = 0$ and hence no $p > p_1^{*-}$ such that $e^{I+}(p, 0) = 0$.

Lemma 9. *Let a strategy profile (e^{I+}, e^{I-}, e^U) and a belief system satisfy the following conditions: (i) e^{I+} is pure; (ii) before fully separating, for any $p > p_1^{*-}$ such that $W^{I+}(p, 0) = W^{I+}(p, \mu)$ and $e^{I+}(p, 0) = 0$, we have $e^{I+}(p, \mu) = 0$; (iii) e^U increases in μ; (iv) the belief system satisfies* (11). *Then at any state (p, μ) that is not fully separating, type s_+ has no incentive to deviate to effort lower than $e^{I+}(p, \mu)$.*

Proof. Let a strategy profile and a belief system satisfy the conditions in Lemma 9. Before fully separating, as long as good news does not arrive and type s_- plays e^{I+}, the informed agent's reputation at t, when the time-t background belief is at p_t, will be $\tilde{\boldsymbol{\mu}}(p_t)$ for some function $\tilde{\boldsymbol{\mu}}$.

Suppose Lemma 9 does not hold. That is, there is some p' such that type s_+ finds it optimal to deviate to a lower effort than equilibrium effort. According to condition (iv), immediately after this deviation type s_+'s reputation will be 0, hence the highest continuation value he can get is $W^{I+}(p', 0)$, which is obtained from type s_+ playing a best response to $e^U(\cdot, 0)$. Therefore, at p', $W^{I+}(p', 0) > W^{I+}(p', \tilde{\boldsymbol{\mu}}(p'))$. Since $W^{I+}(p, 0) \leq W^{I+}(p, \tilde{\boldsymbol{\mu}}(p))$ at the background belief where full separation occurs, and both $W^{I+}(p,0)$ and $W^{I+}(p,\tilde{\boldsymbol{\mu}}(p))$ are continuous in p, there exists some $p'' \in (0,p')$ (before fully separating) such that $W^{I+}(p'',0) = W^{I+}(p'',\tilde{\boldsymbol{\mu}}(p''))$, and that $\frac{dW^{I+}(p''+,0)}{dp} > \frac{dW^{I+}(p''+,\tilde{\boldsymbol{\mu}}(p''+))}{dp}$.

Due to condition (iv), that is, a reputation once lost is lost forever, $W^{I+}(p,0)$ satisfies the following HJB equation:

$$
\begin{aligned}
& rW^{I+}(p,0) \\
= \; & \max_{e\in[0,1]} e\left[r\left(\lambda^{I+}(p)h-s\right)-\lambda p(1-p)\frac{dW^{I+}(p,0)}{dp}+\lambda^{I+}(p)\left(\lambda h-W^{I+}(p,0)\right)\right] \\
& +e^{U}(p,0)\left[-\lambda p(1-p)\frac{dW^{I+}(p,0)}{dp}+\lambda^{I+}(p)\left(\lambda h-W^{I+}(p,0)\right)\right]+rs
\end{aligned}
$$

$W^{I+}(p,\tilde{\boldsymbol{\mu}})$ satisfies the following HJB equation:

$$
\begin{aligned}
& rW^{I+}(p,\tilde{\boldsymbol{\mu}}(p)) \\
= \; & e^{I+}(p,\tilde{\boldsymbol{\mu}}(p))\left[r\left(\lambda^{I+}(p)h-s\right)-\lambda p(1-p)\frac{dW^{I+}(p,\tilde{\boldsymbol{\mu}}(p))}{dp}+\lambda^{I+}(p)\left(\lambda h-W^{I+}(p,\tilde{\boldsymbol{\mu}}(p))\right)\right] \\
& +e^{U}(p,\tilde{\boldsymbol{\mu}}(p))\left[-\lambda p(1-p)\frac{dW^{I+}(p,\tilde{\boldsymbol{\mu}}(p))}{dp}+\lambda^{I+}(p)\left(\lambda h-W^{I+}(p,\tilde{\boldsymbol{\mu}}(p))\right)\right]+rs
\end{aligned}
$$

If $e^{+}(p'',0)>0$, then by $W^{I+}(p'',0) = W^{I+}(p'',\tilde{\boldsymbol{\mu}}(p''))$, and that $\frac{dW^{I+}(p''+,0)}{dp} > \frac{dW^{I+}(p''+,\tilde{\boldsymbol{\mu}}(p''+))}{dp}$, we must have $e^{+}(p'',\tilde{\boldsymbol{\mu}})=1$. But these inequalities, together with condition (iii), contradict with the two HJB equations above. Therefore, $e^{+}(p'',0)=0$, which implies $e^{+}(p'',\tilde{\boldsymbol{\mu}})=0$ by condition (ii). But again, the inequalities $W^{I+}(p'',0) = W^{I+}(p'',\tilde{\boldsymbol{\mu}}(p''))$, that $\frac{dW^{I+}(p''+,0)}{dp} > \frac{dW^{I+}(p''+,\tilde{\boldsymbol{\mu}}(p''+))}{dp}$, and condition (iii), contradict with the two HJB equations above. □

We now check type s_{-}'s and player U's incentive to deviate.

(1) full separation ($p \le p_2^{*-}$). The nontrivial sub-case is when $p_1^{*+} < p \le p_2^{*-}$, and $\mu = \boldsymbol{\mu}^{o}(p)$.

Type s_{-} has no incentive to deviate because, given the updating rule and that player U will choose the same action as he does (because they will play the symmetric MPE corresponding to the information revealed by player I's action), the tradeoff of working or not he faces is exactly the same with that a two-player team with public information s_{-} faces: in both cases, working has the same current flow payoff and twice the "capital gain" as working alone.Since a two-player team finds it optimal to stop if $p < p_2^{*-}$, so does player I of type s_{-}.

The uninformed player has no incentive to deviate because the informed player already perfectly reveals his type and both play the symmetric MPE in the symmetric information game thereafter.

(2) Gradual revelation ($p_2^{*-} < p < p_{gr}$).

The state is along $\hat{\boldsymbol{\mu}}$. Note that given player U's equilibrium strategy and the belief system, whatever deviation that player I plans to employ, the state will be on $\hat{\boldsymbol{\mu}}$ as long as player I hasn't revealed himself. Likewise, given player I's equilibrium strategy, whatever deviation that player U uses, the state will be on $\hat{\boldsymbol{\mu}}$ as long as player I hasn't revealed himself.

We have shown in the previous step that type s_-'s value at (p_2^{*-}, μ) is at most s. Then in the interval $[p_2^{*-}, \min\{p_1^{*-}, p_{gr}\}]$, given the construction of f, type s_- has no incentive to deviate, and that $W^{I-}(p, \hat{\boldsymbol{\mu}}(p)) = s$. In the interval $[\min\{p_1^{*-}, p_{gr}\}, p_{gr}]$ (if nonempty), player U's effort along $\hat{\boldsymbol{\mu}}$ is the same as in the symmetric MPE with public information s_-, since type s_- is indifferent between experimenting and not experimenting in the symmetric MPE, he is also indifferent between experimenting (and hence not revealing) and $e^{S-} \in (0,1)$ (and hence revealing) along $\hat{\boldsymbol{\mu}}$ in the asymmetric information game.

That the uninformed player has no incentive to deviate along $\hat{\boldsymbol{\mu}}$ follows from the following two lemmas.

Lemma 10. *Suppose in the candidate equilibrium constructed in Section §**??**, during gradual revelation, the uninformed player's effort is as in equation* (**??**)*, and that type s_-'s revealing rate y is such that, the associated revealing path $\hat{\boldsymbol{\mu}}$ is a solution to the ODE problem defined by* (20)*,* (21)*, and* (22)*. The player U's value function during gradual revelation is given by equation* (19)*.*

Proof. By our equilibrium construction, trivially, $W^U(p_2^{*-}, \hat{\boldsymbol{\mu}}(p_2^{*-}))$ satisfies equation (19). Now suppose that there is some $p' \in (p_2^{*-}, p_{gr})$ such that equation (19) does not hold at p. Without loss of generality, assume $W^U(p', \hat{\boldsymbol{\mu}}) > s + s - \lambda^U(p', \hat{\boldsymbol{\mu}})\, h$. Then there must exist an interval of p, subset of (p_2^{*-}, p_{gr}), over which $W^U(p, \hat{\boldsymbol{\mu}}) > s + s - \lambda^U(p, \hat{\boldsymbol{\mu}})\, h$, and $\frac{dW^U(p,\hat{\boldsymbol{\mu}})}{dp} > \frac{-\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp}$. As W^U satisfies the HJB equation (17), these two inequalities imply that $W^U(p, \hat{\boldsymbol{\mu}}) < s + s - \lambda^U(p, \hat{\boldsymbol{\mu}})\, h$ for all p in this interval. A contradiction. □

Along $\hat{\boldsymbol{\mu}}$, the revealing strategy of type s_- guarantees the local incentive of player U, that is, if $\hat{\boldsymbol{\mu}}$ is a solution to the ODE problem defined by (20), (21), and (22), and if W^U is given by equation (19) (by Lemma 10), then player U's local incentive condition (18) is satisfied. The following lemma shows that player U does not have a profitable global deviation.

Lemma 11. *Suppose the informed player's strategy and the belief system is as in Section §5, and that type s_-'s revealing rate y is such that, the associated revealing path $\hat{\boldsymbol{\mu}}$ is a solution to the ODE problem defined by* (20)*,* (21)*, and* (22)*, then the uninformed player has no incentive to deviate.*

Proof. Suppose player U has some profitable deviation $\tilde{e}$ starting at some state $(p', \hat{\boldsymbol{\mu}}(p'))$, $p' \in (p_2^{*-}, p_{gr})$, that gives her continuation value $\tilde{W}^U(p', \hat{\boldsymbol{\mu}}(p')) > W^U(p', \hat{\boldsymbol{\mu}}(p'))$. $\tilde{W}^U$ satisfies the HJB equation (17) with e^U replaced by her deviating action. Since $\tilde{W}^U \leq W^U$ at $(p_2^{*-}, \hat{\boldsymbol{\mu}}(p_2^{*-}))$, there must exist some $p'' \in (p_2^{*-}, p')$ such that $\tilde{W}^U(p'', \hat{\boldsymbol{\mu}}(p'')) > W^U(p'', \hat{\boldsymbol{\mu}}(p''))$, and $\frac{d\tilde{W}^U(p'',\hat{\boldsymbol{\mu}}(p''))}{dp} > \frac{dW^U(p'',\hat{\boldsymbol{\mu}}(p''))}{dp}$, but these inequalities, together with the HJB equation (17), imply that $\tilde{W}^U(p'', \hat{\boldsymbol{\mu}}(p'')) < W^U(p'', \hat{\boldsymbol{\mu}}(p''))$. A contradiction. □

The state is along $\boldsymbol{\mu}^o$. If type s_- deviates, whether the deviation starts today or not, he would either reveals himself, or the state variable jumps on the curve $\hat{\boldsymbol{\mu}}$; in both cases, type s_- at most obtains continuation value $W^{S-}(p)$, which is his value of following the equilibrium strategy. Therefore, he has no incentive to deviate.

(3) full pooling ($p > p_{gr}$). Type s_- has no incentive to deviate: on the one hand, player U works harder when type s_- has not revealed himself than when he has revealed already; on the other, one optimal continuation strategy of type s_- after revelation, that is, experimenting with full resource for $p > p_{gr}$, is the same with his equilibrium strategy of not revealing; since type s_- benefits from player U's effort and he will have the same continuation value at p_{gr} whether he deviates now or not, he strictly prefers not to deviate.

The uninformed player has no incentive to deviate because according to Point 4 in Lemma 5, if player U's continuation value at p_{gr} is as specified by equation (19), and that $C(p_{gr}, \hat{\boldsymbol{\mu}}(p_{gr})) > 0$, which is shown by step 1 in Lemma 8, then player U finds it optimal to experiment with full resource for $p > p_{gr}$. (Here we replace the $\underline{p}$ in Lemma 5 by p_{gr}.)

C.4 Dynamics

C.4.1 U's belief about the risky project (proof of Proposition 1)

Proposition 1 is a result of Lemma 12 and Lemma 13.

Lemma 12. *There exists $\tilde{a} \in (1, \infty)$, such that, if $a > \tilde{a}$, then $d\lambda^U\left(p, \hat{\boldsymbol{\mu}}\left(p\right)\right)/dp|_{p=p_2^{*-}} > 0$; if $a < \tilde{a}$, then $d\lambda^U\left(p, \hat{\boldsymbol{\mu}}\left(p\right)\right)/dp|_{p=p_2^{*-}} < 0$.*

Proof. Let $\boldsymbol{q}_2^{*+} : [1, \infty) \to [0, 1]$ be defined by $\boldsymbol{q}_2^{*+}\left(a\right) = \frac{1}{1+(\frac{1}{q_2^*}-1)\frac{1}{a}}$, for $a \in [1, \infty)$, where $\boldsymbol{q}_2^{*+}(a)$ refers to type s_+'s posterior about the risky project, when type s_-'s posterior is q_2^* and the odd ratio is a. Let $\hat{\boldsymbol{\mu}}^*\left(a\right)$ denote the initial value of $\hat{\boldsymbol{\mu}}$ at p_2^{*-}, implied by condition (21), when the odd ratio is a. (The notations $\boldsymbol{q}_2^{*+}$ and $\hat{\boldsymbol{\mu}}^*$ are only used in this proof.)

Using equation (35), (41) (the definition of function D) and (45), we have, $d\lambda^U\left(p+, \hat{\boldsymbol{\mu}}\left(p+\right)\right)/dp|_{p=p_2^{*-}} < 0$ if and only if at $p = p_2^{*-}$,

$$\begin{aligned}
&\lambda\left(1-\boldsymbol{q}_2^{*+}\left(a\right)\right)\frac{\left[W_q^S\left(\boldsymbol{q}_2^{*+}\left(a\right)\right)\boldsymbol{q}_2^{*+}\left(a\right)-\left(w^S\left(\boldsymbol{q}_2^{*+}\left(a\right)\right)-s\right)\right]}{w^S\left(\boldsymbol{q}_2^{*+}\left(a\right)\right)-s} \\
&\qquad -\frac{\lambda}{\hat{\boldsymbol{\mu}}^*\left(a\right)}\frac{\rho_g\left(1-\rho_b\right)}{\rho_b-\rho_g}-\left(\frac{1}{\hat{\boldsymbol{\mu}}^*\left(a\right)}-1\right)\lambda^{I-}\left(p\right) \quad > \quad 0. \qquad (52)
\end{aligned}$$

Define the following functions:

$$\hat{D}_1\left(q\right) \equiv \lambda\left(1-q\right)\frac{\left(W_q^S\left(q\right)q-\left(w^S\left(q\right)-s\right)\right)}{w^S\left(q\right)-s}, \qquad (53)$$

$$\hat{D}_2\left(a\right) \equiv -\frac{\lambda}{\hat{\boldsymbol{\mu}}^*\left(a\right)}\frac{1}{a-1}-\left(\frac{1}{\hat{\boldsymbol{\mu}}^*\left(a\right)}-1\right)\lambda q_2^*. \qquad (54)$$

Then the left-hand side of inequality (52) equals to

$$\hat{D}(a) \equiv \hat{D}_1\left(\boldsymbol{q}_2^{*+}(a)\right) + \hat{D}_2(a). \tag{55}$$

We now show that there is a unique $\tilde{a} \in (1, \infty)$, such that $\hat{D}(a) > 0$ for $a < \tilde{a}$, and $\hat{D}(a) < 0$ for $a > \tilde{a}$, which completes the proof of Lemma 12. The former statement follows directly from the two claims below:

Claim 4. *$\hat{D}(a)$ strictly decreases in a.*

Claim 5. *$\hat{D}\left(a^S\right) > 0$ for the a^S such that $\boldsymbol{q}_2^{*+}\left(a^S\right) = q^S$ (or equivalently, $p_2^{*-} = p^{S+}$); and $\lim_{a\to\infty} \hat{D}(a) < 0$.*

Recall that we construct equilibrium for $p_2^{*-} \geq p^{S+}$, which is equivalent with $a \in [a^S, \infty)$. By continuity of $\hat{D}$, Lemma 5 says that there is a threshold $\tilde{a}$ such that $\hat{D}(a) > 0$ for $a \in (a^S, \tilde{a})$, and $\hat{D}(a) < 0$ for $a \in (\tilde{a}, \infty)$.

Proof of Claim 4. If we show $\hat{D}_1(q)$ strictly decreases in q, and $\hat{D}_2(a)$ strictly decreases in a, then, since $\boldsymbol{q}_2^{*+}(a)$ strictly increases in a, we would have $\hat{D}(a)$ strictly decreases in a.

(i) $\hat{D}_1(q)$ strictly decreases in q.

Replacing w_q^S in the expression of $\hat{D}_1$ by equation (43), we have

$$\hat{D}_1(q) = \lambda\left(\frac{q\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right) - \frac{r}{2\lambda}s}{w^S(q) - s} - \left(\frac{r}{2\lambda}+1\right)\right).$$

Taking derivative with respect to q and rearranging terms, we have

$$\begin{aligned}
\frac{d\hat{D}_1(q)}{dq} &= \lambda\frac{\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right)\left(w^S(q) - s - qw_q^S(q)\right) + w_q^S(q)\frac{r}{2\lambda}s}{\left(w^S(q)-s\right)^2} \\
&= \lambda\frac{\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right)\left(w^S(q) - s - (q - q_2^*)\,w_q^S(q)\right)}{\left(w^S(q)-s\right)^2} \\
&< 0,
\end{aligned}$$

where the second inequality uses equality $\left(\left(\frac{r}{2\lambda}+1\right)\lambda h - s\right)q_2^* = \frac{r}{2\lambda}s$; the third uses the convexity of w^S and that $w^S(q_2^*) = s$.

(ii) $\hat{D}_2(a)$ strictly decreases in a.

Recall the definition of $\hat{D}_2$, we have

$$\hat{D}_2(a) = -\frac{\lambda}{\hat{\boldsymbol{\mu}}^*(a)}\frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\frac{a}{a-1} + \lambda q_2^*$$

Taking derivative with respect to a, and using the value of $\hat{\boldsymbol{\mu}}^*(a)$ (by equation (21)), we have

$$\begin{aligned}
& -\frac{1}{\lambda}\frac{d\hat{D}_2(a)}{da} \\
= \quad & \frac{\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right) + \lambda h \boldsymbol{q}_2^{*+}(a) - \left(w^S\left(\boldsymbol{q}_2^{*+}(a)\right) - s\right) - \lambda h\left(\boldsymbol{q}_2^{*+}(a) - q_2^*\right)}{s - \lambda q_2^* h} \frac{q_2^*}{\left(\boldsymbol{q}_2^{*+}(a)\right)^2}\frac{a}{a-1}\frac{d\boldsymbol{q}_2^{*+}(a)}{da} \\
& -\frac{w^S\left(\boldsymbol{q}_2^{*+}(a)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(a) - q_2^*\right)}{s - \lambda q_2^* h}\frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\frac{1}{(a-1)^2} \\
= \quad & \frac{\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right) - \left(w^S\left(\boldsymbol{q}_2^{*+}(a)\right) - s\right) + \lambda h q_2^*}{s - \lambda q_2^* h}\frac{1}{a(a-1)}(1 - q_2^*) \\
& -\frac{w^S\left(\boldsymbol{q}_2^{*+}(a)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(a) - q_2^*\right)}{s - \lambda q_2^* h}\frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\frac{1}{(a-1)^2} \\
= \quad & \frac{(q - q_2^*)(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right) - \left(w^S\left(\boldsymbol{q}_2^{*+}(a)\right) - s\right) + q_2^* w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right) + \lambda h q_2^*}{s - \lambda q_2^* h}\frac{1}{a(a-1)}(1 - q_2^*) \\
& -\frac{w^S\left(\boldsymbol{q}_2^{*+}(a)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(a) - q_2^*\right)}{s - \lambda q_2^* h}\frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\frac{1}{(a-1)^2}
\end{aligned}$$

Define $\hat{D}_3 \equiv \left(\boldsymbol{q}_2^{*+}(a) - q_2^*\right) w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right) - \left(w^S\left(\boldsymbol{q}_2^{*+}(a)\right) - s\right)$. Since $\frac{a-1}{a}(1 - q_2^*) = 1 - \frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}$ and $\hat{D}_3 > 0$, we have

$$\begin{aligned}
& -\frac{1}{\lambda}\frac{d\hat{D}_2(a)}{da} \\
= \quad & \frac{\hat{D}_3 \boldsymbol{q}_2^{*+}(a)/q_2^* + \boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right) + \lambda h \boldsymbol{q}_2^{*+}(a)}{s - \lambda q_2^* h}\frac{1}{a(a-1)}\frac{q_2^*(1 - q_2^*)}{\boldsymbol{q}_2^{*+}(a)} \\
& -\frac{w^S\left(\boldsymbol{q}_2^{*+}(a)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(a) - q_2^*\right)}{s - \lambda q_2^* h}\frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\frac{1}{(a-1)^2} \\
= \quad & \frac{1}{s - \lambda q_2^* h}\frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\frac{1}{(a-1)^2}\left[\left(\hat{D}_3\frac{\boldsymbol{q}_2^{*+}(a)}{q_2^*} + \left(\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right) + \lambda h \boldsymbol{q}_2^{*+}(a)\right)\right)\left(1 - \frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\right)\right. \\
& \left. - \left(w^S\left(\boldsymbol{q}_2^{*+}(a)\right) - s + \lambda h\left(\boldsymbol{q}_2^{*+}(a) - q_2^*\right)\right)\right] \\
= \quad & \frac{1}{s - \lambda q_2^* h}\frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\frac{1}{(a-1)^2}\left[\hat{D}_3\frac{\boldsymbol{q}_2^{*+}(a)}{q_2^*}\left(1 - \frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\right) + \hat{D}_3\right] \\
= \quad & \frac{1}{s - \lambda q_2^* h}\frac{1}{(a-1)^2}\hat{D}_3 \\
> \quad & 0
\end{aligned}$$

$\square$

Proof of Claim 5. As $a \to \infty$, we have $\boldsymbol{q}_2^{*+}(a) \to 1$, and $\lim_{a\to\infty} \hat{\boldsymbol{\mu}}^*(a) \in (0,1)$ by the initial condition. Therefore, as $a \to \infty$, we have $\hat{D}_1 \to 0$, $\hat{D}_2 \to -\left(\lim_{a\to\infty} \frac{1}{\hat{\boldsymbol{\mu}}^*(a)} - 1\right) \lambda q_2^* < 0$.

If a is such that $\boldsymbol{q}_2^{*+}(a) = q^S$, then we have $\hat{\boldsymbol{\mu}}^*(a) = 1$ by the initial condition. Therefore

$$\begin{aligned}\frac{1}{\lambda}\hat{D}\left(\boldsymbol{q}_2^{*+}(a)\right) &= \frac{1}{\lambda}\hat{D}_1\left(\boldsymbol{q}_2^{*+}(a)\right) - \frac{1}{a-1} \\ &= \left(1-\boldsymbol{q}_2^{*+}(a)\right)\frac{\hat{D}_3}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s} + \frac{\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(a)}{\boldsymbol{q}_2^{*+}(a)}q_2^* - \frac{1}{a-1}\end{aligned} \tag{56}$$

If we show that

$$\frac{\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(a)}{\boldsymbol{q}_2^{*+}(a)}q_2^* - \frac{1}{a-1} > \frac{1}{a}\left[\frac{1}{a-1} - \frac{\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(a)}{\boldsymbol{q}_2^{*+}(a)}q_2^*\right], \tag{57}$$

then we have $\frac{\boldsymbol{q}_2^{*+}(a) w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(a)}{\boldsymbol{q}_2^{*+}(a)}q_2^* - \frac{1}{a-1} > 0$. This inequality, together with $\hat{D}_3 > 0$ and equation (56), imply $\hat{D} > 0$.

We now show inequality (57). First, by the definition of $\boldsymbol{q}_2^{*+}$, we have $\frac{1-\boldsymbol{q}_2^{*+}(a)}{\boldsymbol{q}_2^{*+}(a)}q_2^* = \frac{1}{a}(1-q_2^*)$. Then,

$$\begin{aligned}&\frac{\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(a)}{\boldsymbol{q}_2^{*+}(a)}q_2^* - \frac{1}{a-1} \\ =& \left(\frac{\left(\boldsymbol{q}_2^{*+}(a)-q_2^*\right) w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s} + \frac{\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s}\frac{q_2^*}{\boldsymbol{q}_2^{*+}(a)}\right)\frac{1}{a}(1-q_2^*) - \frac{1}{a-1} \\ =& \left(\frac{\left(\boldsymbol{q}_2^{*+}(a)-q_2^*\right) w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s} + \frac{\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(a)}{\boldsymbol{q}_2^{*+}(a)}q_2^*\right)\frac{1}{a} - \frac{1}{a-1} \\ >& \left[\frac{1}{a-1} - \frac{\boldsymbol{q}_2^{*+}(a)\, w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s}\frac{1-\boldsymbol{q}_2^{*+}(a)}{\boldsymbol{q}_2^{*+}(a)}q_2^*\right]\frac{1}{a}.\end{aligned}$$

The last inequality uses $\frac{\left(\boldsymbol{q}_2^{*+}(a)-q_2^*\right)w_q^S\left(\boldsymbol{q}_2^{*+}(a)\right)}{w^S\left(\boldsymbol{q}_2^{*+}(a)\right)-s} > 1$, and $\frac{1}{a} - \frac{1}{a-1} = \frac{1}{a(a-1)}$. □

□

Lemma 13. *For any $p \in (p_2^{*-}, p_{gr})$ such that $\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp} = 0$, we have $\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp^2} < 0$.*

Proof. First, simple algebra gives us equality $B(p,0) = B(p,\hat{\boldsymbol{\mu}}) + \hat{\boldsymbol{\mu}}\lambda_\mu^U(p,\hat{\boldsymbol{\mu}})$, and $\hat{\boldsymbol{\mu}}\lambda_\mu^U(p,\hat{\boldsymbol{\mu}}) = \lambda^U(p,\hat{\boldsymbol{\mu}}) - \lambda^{I-}(p)$. Recall that $\boldsymbol{q}^-(p)$ is defined as type s_-'s posterior about the risky project when the background belief is p, and that $\lambda^{I-}(p) = \boldsymbol{q}^-(p)\lambda$.

Suppose there exists some $p' \in (p_2^{*-}, p_{gr})$ such that $\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp} = 0$. Taking derivative on both sides of equality (37) with respect to p, at the $p = p'$, and applying $\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp} = 0$, we have

$$
\begin{aligned}
&-\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}B(p,0)\\
=\;&\frac{d\boldsymbol{q}^-(p)/dp}{\lambda p(1-p)}\Bigg[-\lambda\left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-s+\lambda^U(p,\hat{\boldsymbol{\mu}})h\right)\right)\\
&-\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda-\lambda^U(p,\hat{\boldsymbol{\mu}})\right)\frac{dB(p,\hat{\boldsymbol{\mu}})}{d\boldsymbol{q}^-}\Bigg]
\end{aligned}
$$

Since $B(p,\hat{\boldsymbol{\mu}})=s+s-\lambda^U(p,\hat{\boldsymbol{\mu}})-w^S(\boldsymbol{q}^-)$, and $\frac{d\lambda^U(p,\hat{\boldsymbol{\mu}})h}{dp}=0$ at $p=p'$, we have at $p=p'$,

$$
\begin{aligned}
&-\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}B(p,0)\\
=\;&\frac{d\boldsymbol{q}^-(p)/dp}{\lambda p(1-p)}\Bigg[-\lambda\left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-w^S\left(\boldsymbol{q}^-\right)-B(p,\hat{\boldsymbol{\mu}})\right)\right)\\
&+\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda-\lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}\lambda\boldsymbol{q}^-\left(1-\boldsymbol{q}^-\right)w_q^S\left(\boldsymbol{q}^-\right)\Bigg]
\end{aligned}
$$

If $p'\in[p_2^{*-},p_1^{*-}]$, then $w^S(\boldsymbol{q}^-)=s$ and hence $w_q^S(\boldsymbol{q}^-)=0$. Apply inequality (46) and the definition of function D, we have $-\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}|_{p=p'}<0$.

If $p'\in(p_1^{*-},p_{gr}]$, (which is possible only if $(p_1^{*-},p_{gr}]$ is nonempty,) then by the fact that $e^{S-}(q)\in(0,1)$ (that is, in the symmetric MPE under symmetric information, a player is indifferent between experimenting and not experimenting, given that the other player plays the MPE strategy), we have

$$
\lambda\boldsymbol{q}^-\left(1-\boldsymbol{q}^-\right)w_q^S\left(\boldsymbol{q}^-\right)=r\left(\lambda\boldsymbol{q}^-h-s\right)+\lambda\boldsymbol{q}^-\left(\lambda h-W^S\left(\boldsymbol{q}^-\right)\right).
$$

Using this equation to get rid of w^S and w_q^S, we have

$$
\begin{aligned}
&-\frac{d^2\lambda^U(p,\hat{\boldsymbol{\mu}})}{dp^2}B(p,0)\\
=\;&\frac{d\boldsymbol{q}^-(p)/dp}{\lambda p(1-p)}\Bigg[\left(\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})}{\lambda\boldsymbol{q}^-}-1\right)\lambda rs+\lambda\lambda^U(p,\hat{\boldsymbol{\mu}})B(p,\hat{\boldsymbol{\mu}})\\
&+\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda\boldsymbol{q}^--\lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}\left(r\left(\lambda\boldsymbol{q}^-h-s\right)+\lambda\boldsymbol{q}^-\left(\lambda h-w^S\left(\boldsymbol{q}^-\right)\right)\right)\Bigg]\\
=\;&\frac{d\boldsymbol{q}^-(p)/dp}{\lambda p(1-p)}\left[-\frac{\lambda-\lambda^U(p,\hat{\boldsymbol{\mu}})}{1-\boldsymbol{q}^-}\left(\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})}{\lambda\boldsymbol{q}^-}-1\right)rs\right] &(58)\\
<\;&0. &(59)
\end{aligned}
$$

The last inequality uses the fact that the right-hand side of equation (37) equals to 0 when $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp = 0$ (and some algebra).[63] $\square$

C.4.2 The revealing intensity of type s_-

Lemma 14. *In the gradual revelation phase, for $p \in (p_2^{*-}, p_1^{*-})$ such that $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp < 0$, the revealing intensity of the informed player $(1-\hat{\boldsymbol{\mu}})y(p,\hat{\boldsymbol{\mu}})$ strictly decreases in p.*

Proof. Applying equation (15) and U's indifference condition (18), we have, in gradual revelation phase,

$$\begin{aligned}
&\frac{(1-\hat{\boldsymbol{\mu}})\,y(p,\hat{\boldsymbol{\mu}})\,B(p,\hat{\boldsymbol{\mu}})}{e^{I+}(p,\hat{\boldsymbol{\mu}})+e^{U}(p,\hat{\boldsymbol{\mu}})}\\
=\;& r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})\,h-s\right)+\lambda p(1-p)\,hd\lambda^U(p,\hat{\boldsymbol{\mu}})/dp+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-s-\left(s-\lambda^U(p,\hat{\boldsymbol{\mu}})\,h\right)\right)
\end{aligned} \tag{60}$$

Using equation (37) to replace $d\lambda^U(p,\hat{\boldsymbol{\mu}})/dp$, we have

$$\begin{aligned}
&\frac{(1-\hat{\boldsymbol{\mu}})\,y(p,\hat{\boldsymbol{\mu}})\,B(p,\hat{\boldsymbol{\mu}})}{e^{I+}(p,\hat{\boldsymbol{\mu}})+e^{U}(p,\hat{\boldsymbol{\mu}})}\\
=\;& \left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})\,h-s\right)+2\lambda^U(p,\hat{\boldsymbol{\mu}})(\lambda h-s)\right)\frac{B(p,\hat{\boldsymbol{\mu}})}{B(p,0)}
\end{aligned} \tag{61}$$

Using equality $e^{I+}(p,\hat{\boldsymbol{\mu}})=1$ and rearranging terms, we have

$$(1-\hat{\boldsymbol{\mu}})\,y(p,\hat{\boldsymbol{\mu}})\,B(p,0) = \left(1+e^U(p,\hat{\boldsymbol{\mu}})\right)\left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})\,h-s\right)+2\lambda^U(p,\hat{\boldsymbol{\mu}})(\lambda h-s)\right)$$

For $p \in (p_2^{*-}, p_1^{*-})$, we have $e^U = f(p) = \frac{r(s-\lambda^{I-}(p)h)}{\lambda^{I-}(p)(\lambda h-s)} - 1$; also, $B(p,0) = s-\lambda^{I-}(p)\,h$. Therefore,

$$(1-\hat{\boldsymbol{\mu}})\,y(p,\hat{\boldsymbol{\mu}}) = \frac{r\left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})\,h-s\right)+2\lambda^U(p,\hat{\boldsymbol{\mu}})(\lambda h-s)\right)}{\lambda^{I-}(p)(\lambda h-s)} \tag{62}$$

From this equation, if $d\lambda^U/dp \le 0$, then the left-hand side of equation (62), the revealing intensity, strictly decreases as p increases. $\square$

---

[63] To obtain equality (58), notice that

$$\begin{aligned}
&\frac{\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda\boldsymbol{q}^- - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}\left(r\left(\lambda\boldsymbol{q}^-h-s\right)+\lambda\boldsymbol{q}^-\left(\lambda h-w^S(\boldsymbol{q}^-)\right)\right)\\
=\;&\frac{\left(\lambda\boldsymbol{q}^- - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)}{(1-\boldsymbol{q}^-)}\left(r\left(\lambda^U(p,\hat{\boldsymbol{\mu}})\,h-s\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h-w^S(\boldsymbol{q}^-)\right)\right)+\frac{\left(\lambda\boldsymbol{q}^- - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)^2}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}rs
\end{aligned}$$

Using the fact that the right-hand side of equation (37) equals to 0 when $d\lambda^U/dp = 0$, the right-hand side of the equation above equals to

$$-\lambda\lambda^U(p,\hat{\boldsymbol{\mu}})\,B(p,\hat{\boldsymbol{\mu}})+\frac{\left(\lambda\boldsymbol{q}^- - \lambda^U(p,\hat{\boldsymbol{\mu}})\right)^2}{\lambda\boldsymbol{q}^-(1-\boldsymbol{q}^-)}rs.$$

The equality (58) follows.

C.4.3 U's growing pessimism or growing optimism right before full separation

We here give a more detailed argument than in the main text. We do this in three steps.

Step 1. *U's continuation MB of experimentation equals her flow continuation value.* U's continuation value comes from both players' efforts, with I's effort contributing only to the continuation value whereas her own effort also to the flow value:

$$\underbrace{r(W^U - s)}_{\text{flow continuation value}} \quad = \quad e^U[\underbrace{r(\lambda^U h - s)}_{\text{flow MB}} + \text{continuation MB}] + e^I[\text{continuation MB}].$$

Since I takes effort 1 with probability 1 at any state of the gradual revelation phase, and U's total MB is 0 due to her indifference about experimentation, we have

$$\underbrace{r(W^U - s)}_{\text{flow continuation value}} \quad = \quad \text{continuation MB}.$$

Step 2. *U's continuation value can be approximated (up to first order) by her expected continuation value if I's type were public and both players played the symmetric MPE.*[64] Since U is indifferent about experimenting and not experimenting during the gradual revelation phase, we assume that she takes effort 1 during this phase, that is, she matches her effort with the informed player's effort. Under this alternative strategy, in case I is of type s_+, she receives the same payoff as in the symmetric MPE (under symmetric information s_+). In case I is of type s_-, both players equally share the effort load, which is higher than the single-player solution and lower than the cooperative solution; hence each player's continuation value is between the continuation value corresponding to the symmetric MPE solution and to the cooperative solution; since both the continuation value and its derivative with respect to q^- coincides under the symmetric MPE solution and the cooperative solution, each player's continuation value can be approximated by the symmetric MPE solution, in case I is of type s_-.

Step 3. *A drop in q^- reduces U's flow MB relatively more than it reduces U's continuation MB, whereas a drop in q^+ has the reverse effect.*

U's flow MB is the expectation of her ex post flow MBs, $r(\lambda q^+ h - s)$ and $r(\lambda q^- h - s)$, which are linear in her ex post posteriors, (that is, I's posteriors). This means, a mean-preserving spread (henceforth, MPS) of U's belief profile does not change her flow MB.

U's continuation MB, as we will show later, equals her flow continuation value $r(W^U - s)$, and can be approximated by the weighted average of her flow continuation values in the symmetric information benchmark, $r(w^S(q^+) - s)$ and $r(w^S(q^-) - s)$, near the end of the gradual revelation phase. That is, $W^U - s = \mu(w^S(q^+) - s) + (1-\mu)(w^S(q^-) - s)$. Different from the flow MB function, w^S is convex (from KRC-2005), meaning an MPS of U's belief profile increases her continuation MB. Intuitively, the bigger gap between the ex post posteriors q^+ and q^-, the more precise players' information will be after separation, hence the lower chance they will use the inferior project, leading to a higher future value to

[64] That is, U's continuation MB can be approximated by $\mu w^S(\boldsymbol{q} + (p)) + (1-\mu) w^S(\boldsymbol{q} - (p))$.

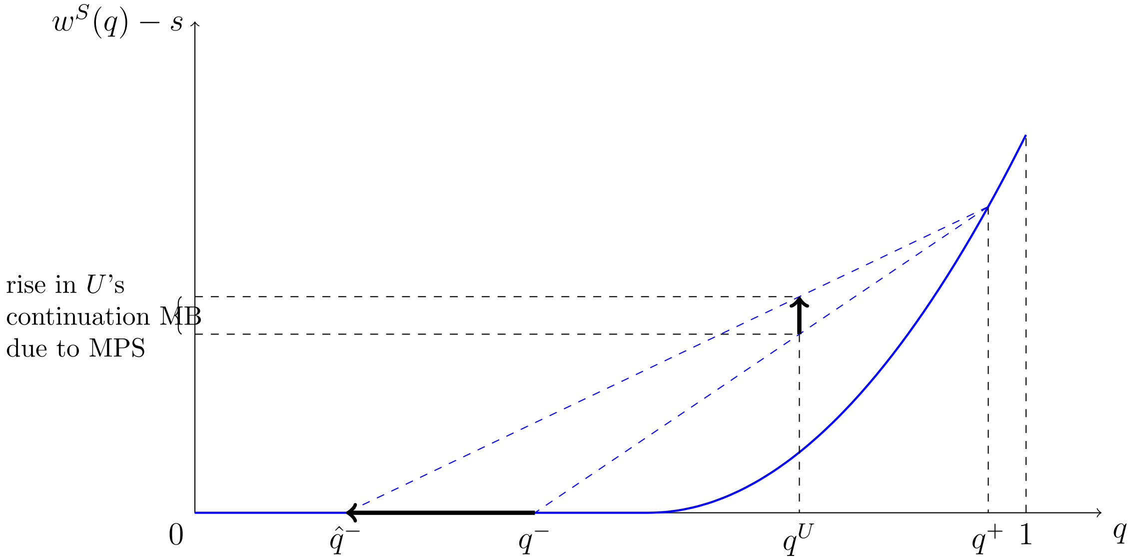

Figure 8: The rise in U's continuation MB due to a mean preserving spread

U, and consequently, a higher incentive for U to accelerate experimentation so as to reap this future value earlier.

A reduction in q^- widens the spread between q^+ and q^- whereas a reduction in q^+ narrows it, resulting in distinct evolution patterns of U's flow MB and her continuation MB, if her total MB were to stay constant. For example, the adjustment — q^+ stays constant, q^- drops by dq, and μ rises by $d\mu$ to keep U's belief about the risky project unchanged — creates an MPS of U's belief profile, whereby it increases her continuation MB without affecting her flow MB. See Figure 8 for an illustration, in which, q^U denotes U's (interim) posterior about the risky project, and $\hat{q}^- = q^- - dq$ type s_-'s posterior after the adjustment; the rise in her continuation MB due to this MPS is represented by the upward pointing arrow. This implies, to keep her total MB constant, I's reputation μ needs to adjust back partially, so that the newly dropping flow MB neutralizes the rising continuation MB. On the contrary, the adjustment — q^- keeps constant, q^+ drops by dq, and μ rises by $d\mu$ to keep U's belief unchanged — makes U's old belief profile an MPS of this new one, whereby it decreases her continuation MB, preserving her flow MB. See Figure 9 for an illustration, in which, $\hat{q}^+ = q^+ - dq$ denotes type s_+'s posterior after the adjustment; the drop in U's continuation MB due to this MPS is represented by the downward pointing arrow. Thus, to keep her total MB constant, μ needs to rise further, so that the newly rising flow MB counterpoises the dropping continuation MB.

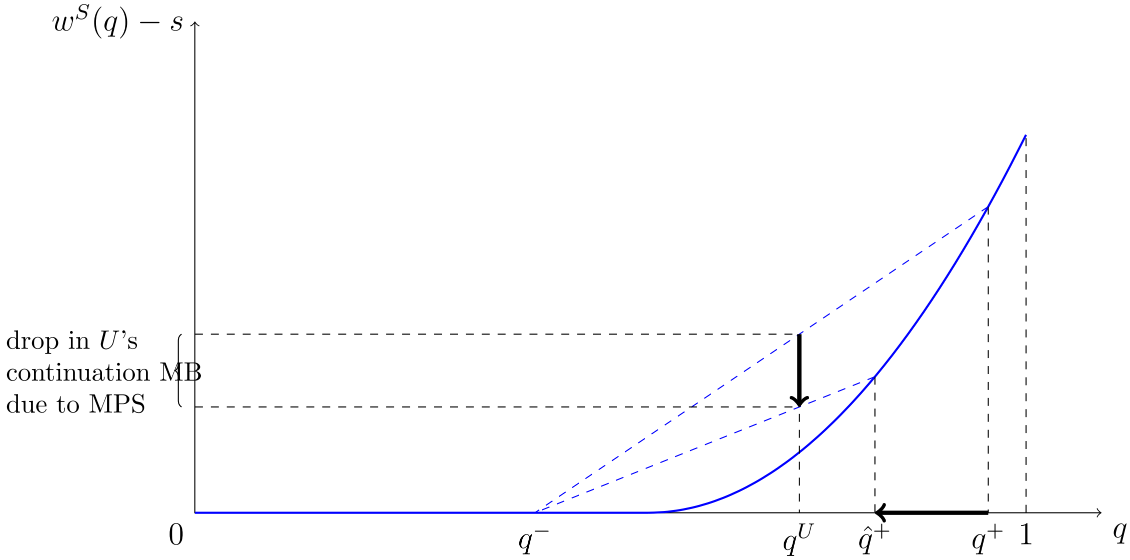

Figure 9: The drop in U's continuation MB due to a mean preserving contraction

C.5 Welfare analysis

C.5.1 Proof of Proposition 1

Depending on the parameters ρ_b and ρ_g, both subcases of case 2 can happen: for instance, consider $a < \hat{a}$ but sufficiently close to $\hat{a}$. If (ρ_b, ρ_g) are sufficiently low, player I will start with a high reputation, implying a short gradual revelation phase, that is, p_{gr} will be close to p_2^{*-}. Hence $\tilde{p}_a = p_{gr}$, and the first subcase of case 2 occurs. If (ρ_b, ρ_g) are sufficiently high, player I will start with a low reputation, implying a long gradual revelation phase, that is, p_{gr} will be far from p_2^{*-}. Hence $\tilde{p}_a < p_{gr}$, and the second subcase of case 2 occurs.

To prove Lemma 1, we first derive an HJB equation for ΔW (in Claim 6), and then show that whenever $\Delta W = 0$ for some $\tilde{p} \in (p_2^{*-}, p_{gr})$, we must have $\Delta W > 0$ for all $p \in (\tilde{p}, p_{gr}]$ (in Claim 7).

Claim 6. *During gradual revelation phase, ΔW satisfies HJB equation* (73)*.*

We first prove this HJB equation holds and then offers an interpretation of it.

Proof of Claim 6. Rewrite the HJB equations of W^U, W^{I+}, W^{S+}, and W^{S-} in the following way:

$$\begin{aligned} & r\left(W^U(p,\hat{\boldsymbol{\mu}}) - s\right) \\ = \; & e^U(p,\hat{\boldsymbol{\mu}})\, r\left[\lambda^U(p,\hat{\boldsymbol{\mu}})\, h - s\right] - (1-\hat{\boldsymbol{\mu}})\, y\left(W^U(p,\hat{\boldsymbol{\mu}}) - W^U(p,\hat{\boldsymbol{\mu}})\right) \\ & + \left(1 + e^U(p,\hat{\boldsymbol{\mu}})\right)\left[-\lambda p(1-p)\frac{dW^U(p,\hat{\boldsymbol{\mu}})}{dp} + \lambda^U(p,\hat{\boldsymbol{\mu}})\left(\lambda h - s - \left(W^U(p,\hat{\boldsymbol{\mu}}) - s\right)\right)\right]. \end{aligned} \quad (63)$$

This is the same with HJB equation (17), except that we collect the two usual optional value parts together: the change in value if no revealing and no good news arrives, and the change in value if no revealing and good news arrives.

$$\begin{aligned} & r\left(W^{I+}(p,\hat{\boldsymbol{\mu}})-s\right) \\ = \;& r\left[\lambda^{I+}(p)\,h-s\right] \\ & +\left(1+e^{U}(p,\hat{\boldsymbol{\mu}})\right)\left[-\lambda p(1-p)\frac{dW^{I+}(p,\hat{\boldsymbol{\mu}})}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(W^{I+}(p,\hat{\boldsymbol{\mu}})-s\right)\right)\right]. \end{aligned} \tag{64}$$

$$\begin{aligned} & r\left(W^{S+}(p)-s\right) \\ = \;& r\left[\lambda^{I+}(p)\,h-s\right] \\ & +\left(1+e^{U}(p,\hat{\boldsymbol{\mu}})\right)\left[-\lambda p(1-p)\frac{dW^{S+}(p)}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(W^{S+}(p)-s\right)\right)\right] \\ & +\left(1-e^{U}(p,\hat{\boldsymbol{\mu}})\right)\left[-\lambda p(1-p)\frac{dW^{S+}(p)}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(W^{S+}(p)-s\right)\right)\right] \\ = \;& r\left[\lambda^{I+}(p)\,h-s\right] \\ & +\left(1+e^{U}(p,\hat{\boldsymbol{\mu}})\right)\left[-\lambda p(1-p)\frac{dW^{S+}(p)}{dp}+\lambda^{I+}(p)\left(\lambda h-s-\left(W^{S+}(p)-s\right)\right)\right] \\ & +\frac{\left(1-e^{U}(p,\hat{\boldsymbol{\mu}})\right)}{2}\left[W^{S+}(p)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]. \end{aligned} \tag{65}$$

The first equality is due to the fact that $p > p^{S+}$, and hence both players experiment with full resource in the symmetric MPE of the symmetric information game. In the second equality, we replace the optional value by $\left[W^{S+}(p)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]/2$, which is obtained from the first equality. The term

$$\left(1-e^{U}(p,\hat{\boldsymbol{\mu}})\right)\left[W^{S+}(p)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]$$

can be interpreted as the welfare loss to both players in case player I is type s_+, caused by the lack of effort of player U (in the equilibrium of asymmetric information game, compared with the symmetric MPE in the symmetric information game).

$$\begin{aligned} & r\left(W^{S-}(p)-s\right) \\ = \;& r\left[\lambda^{I-}(p)\,h-s\right] \\ & +\left(1+e^{U}(p,\hat{\boldsymbol{\mu}})\right)\left[-\lambda p(1-p)\frac{dW^{S-}(p)}{dp}+\lambda^{I-}(p)\left(\lambda h-s-\left(W^{S-}(p)-s\right)\right)\right]. \end{aligned} \tag{66}$$

Writing W^{S-} in this way, we are saying that, in the symmetric information game with public signal s_-, U's payoff is obtained by her experimenting with full resource during $p \in (p_2^{*-}, p_{gr})$ and her teammate experimenting with resource $e^{U}(p,\hat{\boldsymbol{\mu}})$. (Note the role switching between the players.) We may call this a pseudo-equilibrium (as her teammate does not find it optimal to play like this). The reason we interpret W^{S-} in this way is because, the sum of effort will be the same in the equilibrium constructed for the asymmetric information game, and in this pseudo-equilibrium we just specified in the symmetric information game with public signal s_-. This means that in the asymmetric information game, player U enjoys an additional flow payoff $r\left(1-e^U(p,\hat{\boldsymbol{\mu}})\right)\left(s-\lambda^{I-}(p)\,h\right)$ due to effort saving in case I has signal s_-, compared with the symmetric information case.

We now combine these four HJB equations to derive an HJB equation of ΔW.

From equation (24), and replacing $\boldsymbol{\mu}^o$ by $\hat{\boldsymbol{\mu}}$, we have

$$\begin{aligned}\frac{d\Delta W(p,\hat{\boldsymbol{\mu}})}{dp} &= \left(\frac{dW^U(p,\hat{\boldsymbol{\mu}})}{dp}+\hat{\boldsymbol{\mu}}\frac{dW^{I+}(p,\hat{\boldsymbol{\mu}})}{dp}-2\hat{\boldsymbol{\mu}}\frac{dW^{S+}(p)}{dp}-(1-\hat{\boldsymbol{\mu}})\frac{dW^{S-}(p)}{dp}\right)\\ &\quad+\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}\hat{\boldsymbol{\mu}}\left(W^{I+}(p,\hat{\boldsymbol{\mu}})-2W^{S+}(p)+W^{S-}(p)\right) &(67)\\ &= \left(\frac{dW^U(p,\hat{\boldsymbol{\mu}})}{dp}+\hat{\boldsymbol{\mu}}\frac{dW^{I+}(p,\hat{\boldsymbol{\mu}})}{dp}-2\hat{\boldsymbol{\mu}}\frac{dW^{S+}(p)}{dp}-(1-\hat{\boldsymbol{\mu}})\frac{dW^{S-}(p)}{dp}\right)\\ &\quad+\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}\left(\Delta W(p,\hat{\boldsymbol{\mu}})-W^U(p,\hat{\boldsymbol{\mu}})+W^{S-}(p)\right) &(68)\end{aligned}$$

Using equation (**??**), the four HJB equations (63) to (66), we obtain an HJB equation for $\Delta W(p,\hat{\boldsymbol{\mu}})$, with a term

$$\left(\frac{dW^U(p,\hat{\boldsymbol{\mu}})}{dp}+\hat{\boldsymbol{\mu}}\frac{dW^{I+}(p,\hat{\boldsymbol{\mu}})}{dp}-2\hat{\boldsymbol{\mu}}\frac{dW^{S+}(p)}{dp}-(1-\hat{\boldsymbol{\mu}})\frac{dW^{S-}(p)}{dp}\right).$$

We then apply equalities (67) and (68) to get rid of this term, and apply equations (15) and (38) to get rid of $\frac{\hat{\boldsymbol{\mu}}_p}{\hat{\boldsymbol{\mu}}}$. Finally, rearranging terms, we would obtain the HJB equation (73). □

We now interpret the HJB equation of ΔW. To derive a tractable HJB equation of ΔW, we manipulate the value functions (W^{S-} in particular), so that they have a common component $1+e^U(p,\hat{\boldsymbol{\mu}})$ in the optional values. After this manipulation, the effort levels can be interpreted as: in the equilibrium constructed for the asymmetric information game, total effort is $1+e^U(p,\hat{\boldsymbol{\mu}})$, and U's effort is $e^U(p,\hat{\boldsymbol{\mu}})$; in the symmetric information game with public information s_+, total effort is 2, and U's effort is 1; in the symmetric information game with public information s_-, total effort is $1+e^U(p,\hat{\boldsymbol{\mu}})$, and U's effort is 1. Therefore, in the asymmetric information equilibrium, compared with the symmetric information benchmark, total effort is reduced by $1-e^U(p,\hat{\boldsymbol{\mu}})$ in case player I has signal s_-, which causes a reduction in the sum of optional value $\left(1-e^U(p,\hat{\boldsymbol{\mu}})\right)\left[W^{S+}(p)-s-\left(\lambda^{I+}(p)\,h-s\right)\right]$ (see derivation of HJB equation W^{S+} in the proof above); also, in the asymmetric information equilibrium, U's effort is reduced by $1-e^U(p,\hat{\boldsymbol{\mu}})$ (for both cases of signals), saving experimentation cost $\left(1-e^U(p,\hat{\boldsymbol{\mu}})\right)\left[s-\lambda^U(p,\hat{\boldsymbol{\mu}})\,h\right]$. Theses two terms are the first line on the right-hand side of equation (73), resembling the "flow payoff" in a usual HJB equation. The second line on the right-hand side of equation (73), the usual optional value of keeping asymmetric information, is easy to explain: in case good news does not arrive and type s_- does not reveal, ΔW changes by $\frac{d\Delta W(p,\hat{\boldsymbol{\mu}})}{dp}dp$; with probability $\left(1+e^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)dt$, good news arrives, and ΔW jumps to 0; with probability $\left(1-\hat{\boldsymbol{\mu}}\right)y\left(p,\hat{\boldsymbol{\mu}}\right)dt$, type s_{-} reveals his type, and and ΔW jumps to 0 also. In this interpretation, it is important to notice that in the definition of ΔW, type s_{-}'s welfare gain or loss is cancel out, hence such manipulation does not affect type s_{-}'s welfare gain or loss.

Rearranging terms, we have

$$\begin{aligned} & \left(1+e^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)\lambda p\left(1-p\right)\frac{d\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)}{dp} \\ = & -\left[r+\left(1+e^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)+\left(1-\hat{\boldsymbol{\mu}}\right)y\left(p,\hat{\boldsymbol{\mu}}\right)\right]\Delta W\left(p,\hat{\boldsymbol{\mu}}\right) \\ & -\left(1-e^{U}\left(p,\hat{\boldsymbol{\mu}}\right)\right)r\left[\hat{\boldsymbol{\mu}}\left(W^{S+}\left(p\right)-s-\left(\lambda^{I+}\left(p\right)h-s\right)\right)+\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)h-s\right]. \end{aligned} \tag{69}$$

We will show that

Claim 7. *During gradual revelation phase, if there is some $\tilde{p}$ such that $\Delta W\left(\tilde{p},\hat{\boldsymbol{\mu}}\right)=0$, and $\frac{d\Delta W(\tilde{p},\hat{\boldsymbol{\mu}})}{dp}\geq 0$, then $\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)>0$ for all $p\in\left(\tilde{p},p_{gr}\right]$.*

Proof of Claim 7. At $\Delta W\left(p,\hat{\boldsymbol{\mu}}\right)=0$, by using $W^{U}\left(p,\hat{\boldsymbol{\mu}}\right)=s+s-\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)h$, we also have

$$\begin{aligned} & -\left[\hat{\boldsymbol{\mu}}\left(W^{S+}\left(p\right)-s-\left(\lambda^{I+}\left(p\right)h-s\right)\right)+\lambda^{U}\left(p,\hat{\boldsymbol{\mu}}\right)h-s\right] \\ = & \ \hat{\boldsymbol{\mu}}\left(W^{S+}\left(p\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)+\left(1-\hat{\boldsymbol{\mu}}\right)\left(W^{S-}\left(p\right)-s\right) \end{aligned} \tag{70}$$

To prove Claim 7, it is sufficient to show that if the right-hand side of equation (70) is positive at some $\tilde{p}$, then it is positive for all $p\in\left(\tilde{p},p_{gr}\right]$.

First, $d\left(W^{S+}\left(p\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)/dp>0$. This is because, $W^{S+}\left(p\right)$ and $W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)$, when taken as functions of type s^{+}'s posterior $\boldsymbol{q}^{+}(p)$, have derivative w.r.t $\boldsymbol{q}^{+}(p)$ in $[0,\lambda h]$. Hence the derivative of $W^{S+}\left(p\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)$, when taken as function $\boldsymbol{q}^{+}(p)$, w.r.t. $\boldsymbol{q}^{+}(p)$, is in $[-\lambda h,\lambda h]$. Since the derivative of $\lambda^{I+}\left(p\right)h$, when taken as function $\boldsymbol{q}^{+}(p)$, w.r.t. $\boldsymbol{q}^{+}(p)$, is λh, we have that $\left(W^{S+}\left(p\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)$, when taken as function of $\boldsymbol{q}^{+}(p)$, has positive derivative w.r.t. $\boldsymbol{q}^{+}(p)$. As $\boldsymbol{q}^{+}(p)$ strictly increases in p during gradual revelation phase, we have $d\left(W^{S+}\left(p\right)-W^{I+}\left(p,\hat{\boldsymbol{\mu}}\right)+\lambda^{I+}\left(p\right)h-s\right)/dp>0$

Second, the right-hand side of equation (70) is strictly increasing in p whenever it is 0. This is because if at p where it is 0, we have $\left(W^{S+}\left(p\right)-s-\left(\lambda^{I+}\left(p\right)h-s\right)\right)\leq 0$. As $\hat{\boldsymbol{\mu}}$ strictly decreases in p by Proposition 1, and W^{S-} strictly increases in p, the right-hand side of equation (70) is strictly increasing at such p's. $\square$

Therefore, if type s_{+}'s true posterior is above the myopic threshold, then asymmetric information improves welfare. The intuition is the following. In the gradual revelation phase, right before fully separating, U's gain (per unit of time) from asymmetric information in case I holding signal s_{-}, due to the team's high effort, is

$$re^{U}(\lambda^{I-}h-s)+(1+e^{U})(\lambda h-s),$$

which, by applying the formula for e^{U}, equals

$$r(1-e^{U})(s-\lambda^{I-}h).$$

The team's loss (per unit of time) from asymmetric information in case I holding signal s_+, caused by U's low effort, is

$$r(1-e^U)[(\lambda^{I-}h-s)+(W^{S+}-s-(\lambda^{I+}h-s)),$$

where the first part in the square bracket is U's forgone flow benefit per unit saved effort by U, and the second part is the team's forgone optional value per unit saved effort by U.

Hence the team's net gain from asymmetric information right before fully separating, equals

$$r(1-e^U)[(s-\lambda^U h)-\mu(W^{S+}-s-(\lambda^{I+}h-s)),$$

which is the difference between U's expected flow gain from her saved effort and the loss of the team's optional value in case of s_+ due to U's saved effort. During gradual revelation phase, U's expected flow gain from experimentation also equals the required return $r(W^U-s)$, which, right before fully separating, comes only from the required return in case the risky project being good weighted by its probability, $r\mu(W^{S+}-s)$.[65] Therefore, the team's net gain per unit of U's saved effort from asymmetric information equals difference between the required return in case of s_+ and the optional value, which is the flow gain from experimentation in case of s_+ weighted by its probability μ:

$$r\mu(\lambda^{I+}h-s).$$

C.5.2 Welfare analysis in the pooling phase

Once we know the sign of $\Delta W(p,\hat{\boldsymbol{\mu}})$ at p_{gr}, we would know the sign of $\Delta W(p,\hat{\boldsymbol{\mu}})$ at the full pooling Phase, that is, for $p\in(p_{gr},1)$, because the two have the same sign. The intuition is simple; since during full pooling Phase, both players experiment in the same manner as in the symmetric MPE of the symmetric information game, hence the material payoff collected during the full pooling Phase is the same as in the symmetric MPE of the symmetric information game (for the corresponding posteriors), implying that whether asymmetric information improves welfare depends solely on the continuation value of $\Delta W(p,\hat{\boldsymbol{\mu}})$ at p_{gr}.

From Lemma 1 and the above argument, we arrive at the following corollary:

Corollary 1. *There exists $\hat{a}\in(a^S,\infty)$ such that*

1. *If $a\geq\hat{a}$, then $\Delta W(p,\hat{\boldsymbol{\mu}})>0$ over $(p_2^{*-},1)$.*
2. *If $a<\hat{a}$, and*
   (a) *if there exists $\tilde{p}_a\in(p_2^{*-},p_{gr})$ such that $\Delta W(\tilde{p}_a,\hat{\boldsymbol{\mu}})=0$, then $\Delta W(p,\hat{\boldsymbol{\mu}})<0$ over $(p_2^{*-},\tilde{p}_a)$, and $\Delta W(p,\hat{\boldsymbol{\mu}})>0$ over $(\tilde{p}_a,1)$;*
   (b) *otherwise, $\Delta W(p,\hat{\boldsymbol{\mu}})<0$ over $(p_2^{*-},1)$.*

[65] This is obtained from the initial condition (21).

D Further Intuition for Welfare Analysis

As is discussed in the Welfare Analysis section of the main text, Proposition 3 also conveys two other messages. We first give a rough intuition, and then develop it in detail.

First, if asymmetric information improves ex ante total welfare at some background belief p', then so does it at background beliefs higher than p'. This is because the flow part of ΔW strictly increases in the background belief when it is negative, meaning that if ΔW is positive at some background belief p', then it remains its sign over $(p', 1)$. In short, ΔW crosses 0 at most once over $(p_2^{*-}, 1)$.

Second, if right before fully separating, type s_+'s true posterior is at his myopic threshold, then asymmetric information improves ex ante total welfare at all states. This is because if $a \geq a^M$, then asymmetric information improves ex ante total welfare for a small interval of time before full separation (or equivalently, over some right neighborhood of p_2^{*-}), and hence the same is true at all background beliefs, combining the second point.

We now give detailed intuition. As is discussed right before Proposition 3, we only need to analyze welfare over the interval $(p_2^{*-}, p^{S-}]$. Because ΔW evolves differently in sub-case $[p_{gr}, p^{S-}]$ (in the full pooling Phase) and in sub-case (p_2^{*-}, p_{gr}) (in the gradual revelation phase), we analyze each separately, starting from the easier one, the former sub-case.

Step 1. ΔW crosses 0 at most once over $[p_{gr}, p^{S-}]$.

We only focus on the case when $p_{gr} \geq p_1^{*-}$, which means in the symmetric MPE with s_- being public, both players allocate interior resource to the risky project: $e^{S-} \in (0, 1)$ over $[p_{gr}, p^{S-}]$. Analysis for the other case, $p_{gr} < p_1^{*-}$, is qualitatively the same.

Table 1 presents players' efforts in each setup for background beliefs in this interval.

	Asym Info	Sym Info with s_+	Sym Info with s_-
U's effort	1	1	e^{S-}
s_+'s effort	1	1	
s_-'s effort	1		e^{S-}
total effort	2	2	$2e^{S-}$

Table 1: Efforts over $[p_{gr}, p^{S-}]$ in two info setups

We now heuristically explain ΔW satisfies the following HJB equation over $[p_{gr}, p^{S-}]$:

$$\begin{aligned} r\Delta W(p) &= 2r(1-\boldsymbol{\mu}^o)\left(1-e^{S-}(p)\right)\left(s-\lambda^{I-}(p)\,h\right) \\ &\quad +2\left[-\lambda p(1-p)\frac{d\Delta W(p)}{dp}-\lambda^U(p,\boldsymbol{\mu}^o)\,\Delta W(p)\right]. \end{aligned} \tag{71}$$

As can been seen from Table 1, both players experiment as they would do in the symmetric information setup with s_+ public, and more than they would do with s_- public. Therefore, the required return to the team by keeping I's signal private, includes two usual optional value terms on the second line of equation (71): a change of value due to the drop of background belief in case good news does not arrive, and a change of value by an amount

$-\Delta W$ in case good news arrives (because ΔW jumps to 0 should this happen) times the expected arriving rate (per unit of effort and time) λ^U. On top of these two terms, the required return to the team includes an additional gain

$$2r(1-\boldsymbol{\mu}^o(p_0))\left(1-e^{S-}(p)\right)\left(s-\lambda^{I-}(p)\,h\right)$$

per unit of time, generated by the extra amount of effort $(1-e^{S-})$ put by each player, compared with that in the symmetric MPE with s_- public. For ease of explanation, we call this term the "quasi-flow", where "quasi" is used because this extra gain contains both flow and optional value.

Since this "quasi-flow" is strictly positive over $[p_{gr}, p^{S-})$, using backward induction, if ΔW is 0 at some $p' \in [p_{gr}, p^{S-})$, ΔW would be strictly positive over $(p', p^{S-}]$.

Step 2. ΔW crosses 0 at most once over $(p_2^{*-}, p_{gr}]$; $\Delta W > 0$ over $(p_2^{*-}, p_{gr}]$ if and only if $a \geq a^M$.

Over $(p_2^{*-}, p_{gr}]$, in the asymmetric information game, type s_- is indifferent between revealing and not revealing, hence his continuation value are the same in both information setups. $\Delta W(p_0)$ can thus be written as

$$\Delta W(p_0) \;=\; [W^U(p_0,\boldsymbol{\mu}^o)-\boldsymbol{\mu}^o W^{S+}(p_0)-(1-\boldsymbol{\mu}^o)W^{S-}(p_0)]+\boldsymbol{\mu}^o[W^{S+}(p_0)-W^{I+}(p_0,\boldsymbol{\mu}^o)], \tag{72}$$

and be interpreted as the difference between U's gain from asymmetric information (the term in the first square brackets) and type s_+'s loss from asymmetric information (the term in the second). Because the initial state $(p_0, \boldsymbol{\mu}^o)$ jumps immediately to $(p_0, \hat{\boldsymbol{\mu}})$ right after time 0 conditional on non-revealing, it is more convenient to work with the "ex ante" welfare gain after the jump, denoted by $\Delta W\,(p, \hat{\boldsymbol{\mu}})$ for $p = p_0$, which equals the right-hand side of equation (72) with $(p_0, \boldsymbol{\mu}^o)$ replaced by $(p, \hat{\boldsymbol{\mu}})$. It is legitimate to study $\Delta W\,(\cdot, \hat{\boldsymbol{\mu}})$ rather than $\Delta W(\cdot)$ because $\Delta W(p_0)$ is (strictly) positive if and only if $\Delta W\,(p_0, \hat{\boldsymbol{\mu}})$ is (strictly) positive.

We now explain heuristically that $\Delta W(p, \hat{\boldsymbol{\mu}})$ satisfies the following HJB equation:

$$\begin{aligned} & r\Delta W\,(p,\hat{\boldsymbol{\mu}}) \\ = \;& -r\left(1-e^U(p,\hat{\boldsymbol{\mu}})\right)\left[\hat{\boldsymbol{\mu}}\left(W^{S+}(p)-s-\left(\lambda^{I+}(p)\,h-s\right)\right)+\lambda^U(p,\hat{\boldsymbol{\mu}})\,h-s\right] \\ & +\left(1+e^U(p,\hat{\boldsymbol{\mu}})\right)\left[-\lambda p\,(1-p)\,\frac{d\Delta W\,(p,\hat{\boldsymbol{\mu}})}{dp}-\lambda^U(p,\hat{\boldsymbol{\mu}})\,\Delta W\,(p,\hat{\boldsymbol{\mu}})\right]-(1-\hat{\boldsymbol{\mu}})\,y\Delta W\,(p,\hat{\boldsymbol{\mu}})\,. \end{aligned} \tag{73}$$

To understand this HJB equation, it helps to reinterpret $\Delta W(\cdot, \hat{\boldsymbol{\mu}})$ in the following way. First, in the symmetric information game with s_- public, over background beliefs (p_2^{*-}, p_{gr}), if player U plays with an artificial player who plays $e^U\,(\cdot, \hat{\boldsymbol{\mu}}\,(\cdot))$ at the corresponding background beliefs, and U herself experiments with full resource, then U's continuation value would be W^{S-}. This is so because player U in this artificial setup is in exactly the same situation as type s_- is in the gradual revelation phase (over (p_2^{*-}, p_{gr})) in the asymmetric information game, and type s_- receives continuation value W^{S-} as he is indifferent between revealing and not revealing (by fully experimenting) when his teammate plays the equilibrium strategy

$e^U(\cdot, \hat{\boldsymbol{\mu}}(\cdot))$.[66] After this reinterpretation, we present in Table 2 players' efforts in the two information setups:

	Asym Info	Sym Info with s_+	Sym Info with s_-
U's effort	e^U	1	1
s_+'s effort	1	1	
s_-'s (pseudo) effort	1		e^U
total effort	$1+e^U$	2	$1+e^U$

Table 2: Efforts over $[p_2^{*-}, p_{gr}]$ in two info setups

Combining HJB equation (73) and Table 2, we see the second line of equation (73) includes three optional value terms to the team by keeping I's signal private; the first two terms are the same as that in the previous sub-case, $p \in [p_{gr}, p^{S-}]$, and the third term exists due to the fact that in case type s_- reveals, which happens with probability $(1-\hat{\boldsymbol{\mu}})y$ per unit of time, ΔW jumps to 0. On top of these optional value terms, the required return (to the team by keeping I's signal private) includes an additional "quasi-flow" term, on the first line of the right-hand side of equation (73). We now explain this term. From Table 2, compared with symmetric information, in the asymmetric information game, at any instant,

- U works less by $(1-e^U)$, saving her flow experimentation cost

$$r(1-e^U)(s-\lambda^U h);$$

- s_+ works in the same manner, and hence does not experience any flow gain or loss;
- In case I holds signal s_+, due to a lower total effort, the team loses optional value

$$r(1-e^U)[(\lambda^{I-}h-s)+(W^{S+}-s-(\lambda^{I+}h-s))];$$

  in case I holds signal s_-, the team does not experience any optional value gain or loss since total efforts are the same in both information setups.

Summing over these terms together, the "quasi-flow" equals

$$r\left(1-e^U\right)\left[s-\lambda^U(p,\hat{\boldsymbol{\mu}})\,h-\hat{\boldsymbol{\mu}}\left(W^{S+}(p)-s-\left(\lambda^{I+}(p)\,h-s\right)\right)\right]$$

Let's call the terms in the square brackets as the "quasi-flow" per unit of saved effort. It has the following two properties, each of which sheds light on the implication for welfare.

First, the "quasi-flow" per unit of saved effort crosses the line 0 at most once over (p_2^{*-}, p_{gr}), because it is increasing in p whenever it is negative. Hence ΔW remains positive for $p > p'$ if it is positive at some p', during the gradual revelation phase, if we use backward induction.

[66] That is, in the symmetric information game with s_- public, by swapping the roles player U and type s_- play in the asymmetric information setup, U's continuation value (in the symmetric information game with public information s_-) must equal type s_-'s in the asymmetric information game.

Second, the "quasi-flow" per unit of saved effort is positive at p_2^{*-} if and only if the odds ratio is larger than a^M. At p_2^{*-}, right before fully separating, U's continuation value relative to the safe project, $s - \lambda^U(p, \hat{\boldsymbol{\mu}})\, h$, equals $\hat{\boldsymbol{\mu}}\left(W^{S+}(p) - s\right)$, her continuation value relative to the safe project in case I is of type s_+ weighted by its probability. Hence the "quasi-flow" per unit of saved effort equals $\hat{\boldsymbol{\mu}}\left(\lambda^{I+}(p)\, h - s\right)$, which is type s_+'s flow experimentation payoff weighted by I's reputation, and is positive if and only if $a \geq a_M$. Using the first property, we conclude that ΔW is positive during the Gradual Revelation Phase if and only if $a \geq a_M$.

Step 3. Combining the analysis in the previous two steps, we see that if asymmetric information improves ex ante total welfare at some background belief p', then so does it at background beliefs higher than p', and that asymmetric information improves ex ante total welfare at p_2^{*-} if and only if $a \geq a_M$. Therefore, we've explained the last two messages conveyed by Proposition 3.

E Other Equilibria

E.1 Proof of Claim 1

Following the discussion right above Claim 1, we have, if $a > \bar{a}$, then I's continuation value at p_2^{*-} is his continuation value in the symmetric MPE with his private information being public: $W^{I+}(p,\mu) = w^S(\boldsymbol{q}^+(p))$, and $W^{I-}(p,\mu) = w^S(\boldsymbol{q}^-(p))$ $(= s)$, at $p = p_2^{*-}$, and $\mu > 0$.

Let $\hat{\boldsymbol{\mu}} : (p_2^{*-}, p_{gr}) \to [0,1]$ be the gradual revelation path in the constructed MPE. And denote U's effort during the rewarding region of the gradual revelation path as $f(p)$. Note that, given an MPE that coincides with the constructed MPE over background beliefs $[p_2^{*-}, p]$, if I's reputation at p is fixed, then type s_+ strictly prefers to experiment over $[p, p+dp]$, whereas type s_- strictly prefers not to so over the rewarding region or over the non-responding region if the uninformed player's effort is strictly higher than $e^S(\boldsymbol{q}^-)$; over $[p, p+dp]$, the uninformed player strictly prefers to experiment for $\mu > \hat{\boldsymbol{\mu}}$, and be willing to experiment at $\mu < \hat{\boldsymbol{\mu}}$ only if the informed player's equilibrium effort strictly lower than 1. [Recall again that players' current efforts are strategic substitutes; the uninformed player is willing to experiment at lower beliefs only if the informed player's effort is lower, before separating].

We will use backward induction to show that, in any MPE, if the equilibrium strategies over $[p_2^{*-}, p]$ is the same as in the constructed MPE, then the equilibrium strategy over $[p, p+dp]$ in the former MPE will be the same as in the constructed MPE.

Proof. Note that in the constructed MPE, $\hat{\boldsymbol{\mu}}$ is the borderline such that, if I's effort is 1, then U strictly prefers to experiment if the state is above it, and strictly prefers not to if the state is below it, and is indifferent if the state is on this curve. Hence below this curve, U is willing to experiment only if I's effort is lower than 1.

Consider an MPE of the asymmetric information game, and denote the equilibrium effort strategies of the uninformed player and of type s_+ as $\tilde{e}^U$, $\tilde{e}^{I+}$. Let $\tilde{p}$ be the infimum over $[p_2^{*-}, p_{gr})$ such that equilibrium strategies differ from the constructed MPE.

(1) If there is some $\mu < \hat{\boldsymbol{\mu}}$ such that the equilibrium strategies differ from the constructed MPE over $[\tilde{p}, \tilde{p} + dp]$, then let $\tilde{\mu}$ be such that the averaged uninformed player's effort is the lowest over $[\tilde{p}, \tilde{p} + dp]$. For type s_- to mimic type s_+, we must have either, $\tilde{e}^U/\tilde{e}^{I+} \geq f(p)$ over the rewarding region, or $\tilde{e}^U \geq e^S(\boldsymbol{q}^-(p))$ over the non-responding region, both requiring $\tilde{e}^{I+} < 1$ (otherwise U would strictly prefer not to experiment, given that μ is low). If I takes action $\tilde{e}^{I+}$, then at $\tilde{p}$, he will end up at state $(\tilde{p}, \hat{\boldsymbol{\mu}})$ (since the equilibrium over $[p_2^{*-}, \tilde{p}]$ coincides with the constructed MPE). Now consider I deviating to effort 1 at reputation $\tilde{\mu}$ and over the interval $(\tilde{p}, \tilde{p}+dp)$. Type s_+ strictly benefits from such a deviation as long as he does not get a perfect bad reputation: he obviously gains if his reputation reaches above $\hat{\boldsymbol{\mu}}$; if he gets a reputation below $\hat{\boldsymbol{\mu}}$ (but still positive), then at $\tilde{p}$ his reputation immediately jumps upward to $\hat{\boldsymbol{\mu}}$ after taking his equilibrium strategy $\tilde{e}^{I+}(=1)$, hence he also benefits. But type s_- strictly loses if his reputation does not change (hence jumps upward to $\hat{\boldsymbol{\mu}}$ after taking action $\tilde{e}^{I+}(=1)$ at $\tilde{p}$). That is, the set of reputation making type s_+ strictly benefit from such a deviation is strictly larger than the set of reputation making type s_- weakly benefit from it. Therefore, by D1, after such a deviation, I should receive a perfect reputation; but this suggests that type s_- strictly prefers to deviate to effort 1 over $(\tilde{p}, \tilde{p}+dp)$ at reputation $\tilde{\mu}$.

(2) If the equilibrium strategies differ from the constructed MPE over $[\tilde{p}, \tilde{p}+dp]$, only at reputations $\mu \geq \hat{\boldsymbol{\mu}}$, which is possible only if $\tilde{e}^{I+} < 1$ and $\tilde{e}^U = 1$. Consider I deviating to effort 1 at such a reputation μ, over the interval $(\hat{p}, \hat{p} + dp)$. Then type s_+ strictly benefits from such a deviation as long as he receives a reputation weakly above μ, whereas type s_- strictly loses if he gets a reputation weakly below μ; therefore, the set of reputation making type s_+ strictly benefit from such a deviation is strictly larger than the set of reputation making type s_- weakly benefit from it. By D1, after such a deviation, I should receive a perfect reputation; but this suggests that type s_- strictly prefers to deviate to effort 1 over $(\tilde{p}, \tilde{p} + dp)$ at reputation μ.

Therefore, any MPE satisfying D1 should coincide with the constructed MPE over $[p_2^{*-}, p_{gr}]$ (the gradual revelation region); that such MPE coincides with the constructed MPE over the pooling region follows similar steps. $\square$

References

Aghion, Philippe, and Peter Howitt. 1992. "A Model of Growth through Creative Destruction." *Econometrica*, 60(2): 323–51.

Aumann, Robert J. 1964. "Mixed and behavior strategies in infinite extensive games." In *Advances in Game Theory. Annals of Mathematics*, , ed. Melvin Dresher, L.S. Shapley and A.W. Tucker, 627–650. Princeton University Press.

Avolio, Bruce J., and William L. Gardner. 2005. "Authentic leadership development: Getting to the root of positive forms of leadership." *The Leadership Quarterly*, 16(3): 315 – 338. Authentic Leadership DevelopmentGetting to the Root of Positive Forms of Leadership.

Bandiera, Oriana, and Imran Rasul. 2006. "Social Networks and Technology Adoption in Northern Mozambique*." *The Economic Journal*, 116(514): 869–902.

Banerjee, Abhijit V. 1992. "A Simple Model of Herd Behavior." *The Quarterly Journal of Economics*, 107(3): 797–817.

Bass, B.M., and R. Bass. 2009. *The Bass Handbook of Leadership: Theory, Research, and Managerial Applications.* Free Press.

Bolton, Patrick, and Christopher Harris. 1999. "Strategic Experimentation." *Econometrica*, 67(2): 349–374.

Bonatti, Alessandro, and Johannes Hörner. 2011. "Collaborating." *American Economic Review*, 101(2): 632–63.

Carrillo, Juan D., and Thomas Mariotti. 2000. "Strategic Ignorance as a Self-Disciplining Device." *The Review of Economic Studies*, 67(3): 529–544.

Coleman, James, Elihu Katz, and Herbert Menzel. 1957. "The Diffusion of an Innovation Among Physicians." *Sociometry*, 20(4): 253–270.

Conley, Timothy G., and Christopher R. Udry. 2010. "Learning about a New Technology: Pineapple in Ghana." *American Economic Review*, 100(1): 35–69.

Crémer, Jacques. 1995. "Arm's Length Relationships." *The Quarterly Journal of Economics*, 110(2): 275–295.

Das, Kaustav. 2015. "Strategic Experimentation with Competition and Private Arrival of Information."

Doz, Yves L. 1996. "The evolution of cooperation in strategic alliances: Initial conditions or learning processes?" *Strategic Management Journal*, 17(S1): 55–83.

Foster, Andrew D., and Mark R. Rosenzweig. 1995. "Learning by Doing and Learning from Others: Human Capital and Technical Change in Agriculture." *Journal of Political Economy*, 103(6): 1176–1209.

Gardner, William L., Bruce J. Avolio, Fred Luthans, Douglas R. May, and Fred Walumbwa. 2005. ""Can you see the real me?" A self-based model of authentic leader and follower development." *The Leadership Quarterly*, 16(3): 343 – 372. Authentic Leadership DevelopmentGetting to the Root of Positive Forms of Leadership.

Guo, Yingni, and Anne-Katrin Roesler. 2016. "Private Learning and Exit Decisions in Collaboration." Northwestern University Working Paper.

Heidhues, Paul, Sven Rady, and Philipp Strack. 2015. "Strategic experimentation with private payoffs." *Journal of Economic Theory*, 159, Part A: 531 – 551.

Hermalin, Benjamin E. 1998. "Toward an Economic Theory of Leadership: Leading by Example." *American Economic Review*, 88(5): 1188–1206.

Hörner, Johannes, Nicolas Klein, and Sven Rady. 2014. "Strongly Symmetric Equilibria in Bandit Games." Free University of Berlin, Humboldt University of Berlin, University of Bonn, University of Mannheim, University of Munich Discussion Paper Series of SFB/TR 15 Governance and the Efficiency of Economic Systems 469.

Iyengar, Raghuram, Christophe Van den Bulte, and Thomas W. Valente. 2011. "Opinion Leadership and Social Contagion in New Product Diffusion." *Marketing Science*, 30(2): 195–212.

Keller, Godfrey, Sven Rady, and Martin Cripps. 2005. "Strategic Experimentation with Exponential Bandits." *Econometrica*, 73(1): 39–68.

Komai, Mana, Mark Stegeman, and Benjamin E. Hermalin. 2007. "Leadership and Information." *The American Economic Review*, 97(3): 944–947.

March, James G., and Thierry. Weil. 2009. *On Leadership.* Wiley.

Nair, Harikesh S., Puneet Manchanda, and Tulikaa Bhatia. 2010. "Asymmetric Social Interactions in Physician Prescription Behavior: The Role of Opinion Leaders." *Journal of Marketing Research*, 47(5): 883–895.

Nelson, Richard R., and Sidney G. Winter. 1994. *An Evolutionary Theory of Economic Change.* Cambridge (Mass.); London:The Belknap Press of Harvard University Press.

Romer, Paul M. 1990. "Endogenous Technological Change." *Journal of Political Economy*, 98(5): S71–102.

Romer, Paul M. 1994. "The Origins of Endogenous Growth." *Journal of Economic Perspectives*, 8(1): 3–22.

Scharfstein, David, and Jeremy Stein. 1990. "Herd Behavior and Investment." *American Economic Review*, 80(3): 465–79.

Yukl, Gary A. 2010. *Leadership in Organizations.* Pearson.